Supply Chain Management and Aggregate Fluctuations

Le Xu*

University of Pennsylvania

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Abstract

Over the business cycle, firms adjust not only their input expenditures but also the number of suppliers from which they source. I incorporate this extensive margin into a real business cycle model. Using a dataset of supply chain relationships among US firms, I first document that increases in the number of suppliers are correlated with increases in intermediate input expenditures, total factor productivities, and costs of managing suppliers. Based on these facts, I develop a model in which firms trade off the productivity benefit (return to variety) of accessing more varieties with the (fixed) cost of managing these varieties. The extensive margin adjustment introduces a return to scale into production and amplifies productivity shocks: In my estimated model with multiple industries and a production network, the effect of industry productivity shocks on GDP fluctuations is one-fourth larger than in a (conventional) model where the extensive margin is absent.

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*Le Xu: Department of Economics, University of Pennsylvania, 133 South 36th Street, Philadelphia, PA 19104 (e-mail: lexu1@sas.upenn.edu) and Research Department, Federal Reserve Bank of Philadelphia, Ten North Independence Mall W, Philadelphia, PA 19106. I am extremely grateful to my advisor Jesús Fernández-Villaverde for his continuous support and guidance, to my committee: Dirk Krueger, Harold L. Cole, Alessandro Dovis, and Simon Freyaldenhoven, and to Enghin Atalay. I also thank David Baqaee, Vasco Carvalho, Xiang Fang, Zhen Huo, Zhesheng Qiu, Alireza Tahbaz-Salehi, and attendants at the Macro seminar, Macro lunch, and Macro student lunch of the Department of Economics at the University of Pennsylvania, and the lunch seminar of the Federal Reserve Bank of Philadelphia for their useful comments. The views expressed in this paper are those of the author and do not necessarily reflect those of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.

1 Introduction

In neoclassical models with intermediate inputs (Acemoglu et al., 2012; Atalay, 2017; Baqaee, 2018; Long Jr. and Plosser, 1983), firms always source inputs from all suppliers. Over the business cycle, however, firms adjust not only their input expenditures but also the number of suppliers from which they source. According to the FactSet supply chain dataset, a median US firm adjusts the number of its suppliers in 71% of the years from 2003 to 2018.

In this paper, I study the adjustment of input variety numbers and incorporate this extensive margin adjustment (EMA) into a real business cycle model.[1] Using a dataset of supply chain relationships among US firms, I first document that increases in intermediate input expenditures are correlated with increases in the number of suppliers. Furthermore, I document that increases in the number of suppliers are correlated with increases in the total factor productivities and the costs of managing suppliers.

Based on these facts, I develop a real business cycle model in which firms trade-off between the productivity benefit of more varieties and the cost of managing varieties to adjust on the extensive margin. The extensive margin and the associated productivity gain introduce a return to scale into production and amplify productivity shocks. In the estimated model with multiple industries and a production network among them, industry productivity shocks generate a real GDP standard deviation of 1.88% with EMA, which is one-fourth larger than that without EMA. In other words, the extensive margin adjustment enables industry productivity shocks to explain a larger share of the aggregate fluctuation in the data.

My paper is motivated not only by the continuous efforts of economists to explain economic fluctuations (Ramey, 2016) but also by observations of extensive margin adjustments in the data. Figure 1a shows that Ford adds suppliers when sales value increases and cuts suppliers when sales value falls. Meanwhile, a similar pattern exists in the number of input varieties (Figure 1b).[2] When Ford's sales started to grow in 2010, it introduced a new navigation system supplier (TeleNav) in addition to the existing supplier (Garmin). With this additional supplier, Ford developed a map-based navigation system for which Ford charged $ 1,000. As a result, Ford became more productive in the sense that it produced cars with higher quality. On the other hand, when its sales slowed down in 2014, Ford abandoned Garmin.

It turns out that the case of Ford reflects a general pattern of adjustments in supplier numbers. By combining the FactSet data on supply chain relationship among US firms with industry-level data from the US Bureau of Economic Analysis (BEA), I document the following three facts about extensive margin adjustments.

[1] In the data, I use the number of suppliers to measure the number of input varieties.

[2] The numbers of suppliers and input varieties are calculated using the FactSet Revere database. Ford's (US) sales values are from Ford's press release and annual reports at https://media.ford.com.

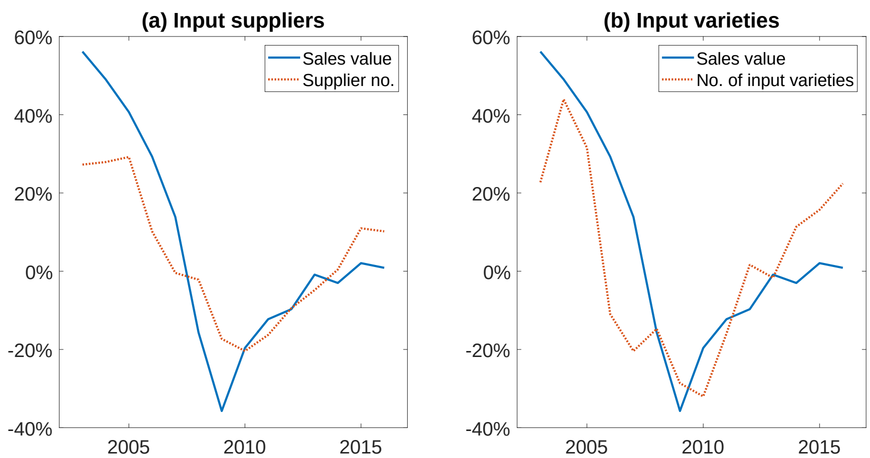

Figure 1: Ford's Sales Value vs. Number of Input Suppliers/Varieties in the US

Note: Y-axes are percentage deviations from the median of each series. Panels (a) and (b) plot Ford's number of suppliers and input varieties vs. sales value, respectively. An input variety is defined as a unique sector of level five in the FactSet hierarchy of sectors. The number of input varieties is calculated as the number of unique varieties to which at least one product produced by any of its supplier belongs.

First, the number of an industry's suppliers increases with its intermediate input expenditures.[3] I find that a 1% increase in an industry's intermediate input expenditures is associated with a 0.28% increase in its number of suppliers. The positive correlation indicates that firms adjust intermediate input expenditures through both the intensive and the extensive margins.[4]

Second, an industry's total factor productivity increases with its number of suppliers. Controlling for intermediate input expenditures, labor input, and capital stock, a 1% increase in the number of a customer industry's input suppliers is associated with a 0.035% increase in its real output. This total factor productivity gain due to more suppliers is a return to (input) variety in production. The return to variety can be interpreted as a reduced-form productivity benefit due to a higher quality of output when more input varieties are used.[5] Moreover, I use the Olley-Pakes and Levinsohn-Petrin methods to estimate firms' production functions, which control for unobserved productivity shocks. I find that a 1% increase in a firm's total number of suppliers is associated with a 0.06% to 0.09% increase in its real sales, controlling for intermediate inputs, capital stock, and the number of employees. Thus, the observed productivity benefit of more suppliers is not because the number of suppliers proxies unobserved productivity shocks.

Third, the cost of managing suppliers increases with the number of suppliers. I proxy the cost

[3]Correlations in this paper are all based on log-differenced regressions and correspond to business cycle variations.

[4]Li (2013) finds a similar result for consumption: consumers increase consumption by increasing both the quantity of each variety and the number of varieties.

[5]Indeed, the source of the productivity gain in my paper is not limited to this example. Extensive margin adjustments with alternative reduced-form productivity gains also amplify industry productivity shocks and aggregate fluctuations.

of a customer industry's supply chain management with the employment number of purchasing agents in this industry. I find that a 1% increase in the supply chain management cost of a customer industry is associated with a 0.27% increase in this industry's total number of suppliers. In contrast, input expenditures are not positively correlated with management costs. These two observations together imply a fixed cost of managing suppliers.

Based on these three facts, I use a real business cycle model featuring the productivity benefit and management cost of input varieties to study the aggregate effects of extensive margin adjustments. In this economy, there is a continuum of differentiated firms, each of which produces a unique variety of goods using inputs from other firms in the economy. When sourcing inputs, a firm chooses both the number of input varieties and the quantity of each variety.[6] Using more input varieties increases the productivity of a firm. However, to use each variety, a firm needs to hire one unit of management labor supplied by the household. The management labor is separated from the production labor and is elastically supplied, which leads to a supply chain management cost that is increasing and convex in the number of input varieties.

The trade-off between the return to (input) variety and the cost of managing varieties leads to extensive margin adjustments, which, in turn, amplify aggregate fluctuations. In booms, firms use more intermediate inputs, and the increase in input quantity per variety raises the marginal return to input varieties. As a result, firms pay higher management costs to use more varieties. A larger variety of inputs, in turn, generates productivity gains and increases output. In busts, the opposite is true. Intuitively, the extensive margin, together with the return to variety, introduces a return to scale into production and amplifies productivity shocks, as well as aggregate fluctuations.

How much the aggregate fluctuation is amplified by extensive margin adjustments depends on the structure of supply chains, in particular, the supply chains among different industries' firms. As a result, I introduce multiple industries and the production network into the quantitative model. To be more specific, the multi-industry real business cycle model has an input-output network among industries. Within each industry, there is a continuum of differentiated firms, each of which produces a unique variety of goods using inputs from a set of firms in different industries.

It turns out that the structure of the production network affects the amplification effect of EMAs. Imagine a vertical economy where manufacturing sources inputs from agriculture, and agriculture sources inputs from utilities. When manufacturing uses more input varieties from agriculture, its productivity, and thus its sales, increase. With a higher demand from manufacturing, agriculture uses more intermediate inputs and thus more input varieties from utilities. In this way, the extensive margin adjustments on different linkages of the same supply chain complement and reinforce each

[6] I make assumptions to focus on the equilibrium in which the number of chosen varieties matters, rather than the idiosyncratic features of the varieties.

other. As a result, the lengths of the supply chains, which depend on the structure of the production network, affect the size of the amplification effect of EMA.

A multi-industry framework also allows me to match industry-level data to estimate the key parameters of the model, which is critical to quantifying the effects of extensive margin adjustments on aggregate fluctuations. In particular, I use indirect inference estimation to estimate the curvature of the disutility function of management labor (in turn, the management cost function) and the input elasticity of substitution between industries. I also calibrate the return to variety by matching the model's steady-state management cost share in total revenue with that in the American Productivity & Quality Center[7] (APQC) survey data. In addition, the calibration of industry productivities, which are computed as Solow residuals, needs to exclude the productivity gain from EMA. Because EMA is endogenous and simulated in indirect inference, the industry productivities are calibrated within the estimation.

With the multi-industry model estimated and calibrated, I quantitatively evaluate EMA's amplification of aggregate fluctuations. In the baseline model with EMA and the production network, the real GDP standard deviation is 1.88%, compared to a standard deviation of 1.51% without EMA. Thus, EMA amplifies the aggregate fluctuation (generated by industry productivity shocks) by one-fourth, which is about 54% of the contribution of labor inputs (hours). I also find that EMA amplifies aggregate fluctuations even if the production and management labor markets are unified, as long as the supply of the homogeneous labor is elastic enough. Finally, I find that shortening the supply chains in the production network reduces EMA's amplification effect.

Related Literature

This paper is related to four strands of the literature. First, it is related to the literature that studies amplification mechanisms that allow small variations in productivity to generate large fluctuations in aggregate outputs. In other words, these mechanisms can account for endogenous variations in measured TFP (Bai et al., 2012; Greenwood et al., 1988; Jaimovich and Floetotto, 2008; Kydland and Prescott, 1988). This literature focuses on varying capital and labor utilization and demand shocks. In comparison, my paper studies the choice of input variety numbers. My paper is closely related to Jaimovich and Floetotto (2008), which finds that the interaction between firms' entry and exit decisions and markups amplifies exogenous productivity shocks. They find this interaction to account for 40% of the variation in measured TFP in the US. Instead of the entry and exit of firms, my paper finds the endogenous input variety choice to be an amplification mechanism. In terms of modeling, my paper is closely related to the work of Huo and Rios-Rull (2019), which is an

[7] www.apqc.org.

extension of the work of Bai et al. (2012). In Huo and Rios-Rull (2019), consumers increase the number of consumption varieties as well as the consumption of each variety when they raise their consumption. My model studies similar adjustments on the production side, i.e., firms increase the number of input varieties and the quantity of each variety when they use more intermediate inputs.

This paper is also related to the international trade literature that finds a productivity gain to an increase in imported intermediate input varieties following the theoretical work by Ethier (1982) (Feenstra et al., 1999; Gopinath and Neiman, 2014; Halpern et al., 2015). Among them, my paper is most related to Gopinath and Neiman (2014) in the sense that we both emphasize the importance of the within-firm choice of input varieties instead of the entry and exit of firms. Indeed, the sub-extensive margin in their paper is exactly the extensive margin in my paper. Compared to their papers, my paper documents a similar productivity gain to a larger number of domestic intermediate input varieties and studies its implication on business cycle fluctuations.

Third, this paper contributes to a growing literature on the importance of production networks in aggregating microeconomic shocks into macroeconomic fluctuations (Acemoglu et al., 2016, 2012; Atalay, 2017; Baqaee, 2018; Huo et al., 2019; Long Jr. and Plosser, 1983). Among them, this paper is related to the work building on the multi-industry business cycle model by Long Jr. and Plosser (1983). Atalay (2017) and Baqaee (2018) argue that industry-specific productivity shocks, propagating in production networks, can generate large aggregate fluctuations. My paper is close to Baqaee (2018), who considers the entry and exit of firms within industries as a propagation and amplification mechanism. My paper, in comparison, studies firms' choice of input variety numbers. The amplification mechanisms in our papers are similar in spirit but rely on different margins. Also, both the empirical facts and the quantitative evaluation of the model indicate that firms' choice of input variety numbers is an important margin for shock transmission and aggregate fluctuations. In addition, the structure of the production network greatly determines the size of the amplification effects of the extensive margin adjustments in my paper. My paper is also related to Atalay (2017), who documents a less-than-unit input elasticity of substitution between different industries' goods. He shows that the small elasticity makes industry-specific shocks substantially more important in generating aggregate fluctuations than those in unit-elasticity models. Following Atalay (2017), I estimate the elasticity of substitution to be less than unity using indirect inference.

Finally, this paper is also related to the recent literature on endogenous supply chains in general equilibrium models (Acemoglu and Azar, 2019; Carvalho and Voigtländer, 2015; Huneeus, 2018; Oberfield, 2018; Taschereau-Dumouchel, 2019; Zou, 2019). Among these models, my paper falls into the category of costly supply chain relationships. In particular, my paper is closely related to Lim (2018) and Taschereau-Dumouchel (2019), who also study aggregate fluctuations but in a firm-level endogenous production network. In these two papers, the aggregate fluctuation is dampened by the option of choosing customers/suppliers through two channels: the substitution between input varieties and the substitution between production and management labor. In my model, I focus on the substitution between industries rather than between varieties. The former one is smaller than the latter one because inputs are more complementary across industries than across firms. In addition, I allow production and management labor to be elastically supplied in segmented markets, which relaxes the one-to-one substitution between the two types of labor. Due to these two features, extensive margin adjustments in my paper amplify rather than dampen aggregate fluctuations.

The remainder of the paper is organized as follows. Section 2 presents the three facts about extensive margin adjustments. Section 3 develops a real business cycle model with EMAs and shows how EMAs amplify aggregate fluctuations. Section 4 introduces the full model with both EMAs and a production network. I use a simplified static model to illustrate the role of the production network in the effects of EMAs on aggregate fluctuations. Section 5 discusses the identification strategy and presents the estimation and simulation results of the full model in Section 4. Section 6 shows the results in a model with a unified market of production and management labor or with alternative calibrations of the extensive margin. Section 7 concludes.

2 Facts about Extensive Margin Adjustments

In this section, I describe the data and present three facts about extensive margin adjustments. First, a 1% increase in the customer industry's total intermediate input expenditure is associated with a 0.281% increase in its total number of suppliers. Second, a 1% increase in the customer industry's input supplier number is associated with a 0.035% increase in its real output. Third, a 1% increase in a customer industry's supply chain management cost is associated with a 0.27% increase in its number of suppliers. Fact one indicates that firms adjust intermediate input expenditures through the number of suppliers. Facts two and three imply the productivity benefit and management cost of input varieties (suppliers).

2.1 Data

Four different datasets are used in the empirical analysis of this section, FactSet Revere, Compustat, CRSP/Compustat Merged, the BEA Input-Output Tables and Industry Accounts, and the Occupational Employment Statistics by the BLS. FactSet Revere gathers supplier-customer relationship data among US firms, and Compustat gathers firms' industry classifications. I use these two datasets to construct producers' numbers of suppliers on the industry level. The facts I document are mainly based on industry-level regressions because the data of input-output network, intermediate input expenditures, real output, purchasing agents' employment, and real output are available on the industry level but not on the firm level. I use industry-level intermediate input expenditures, capital and labor inputs, and real output from the BEA Input-Output Tables and Industry Accounts. I also construct industry-level employment of purchasing agents from the OES. On the firm level, I rely on CRSP/Compustat Merged dataset for firms' real sales, intermediate inputs, capital stock, and labor input. For all the regressions, I restrict the sample to the period from 2003 to 2016, for which the supply chain relationship and the industry-level occupational employment data are both available. Details of the data used are introduced in Appendix A.

2.2 Input Expenditures and Extensive Margin Adjustments

This section presents the facts that industries/firms adjust their numbers of suppliers together with intermediate input expenditures and that the extensive margin adjustment is as significant as the intensive margin. The following fact states the positive correlation between the extensive margin and the input expenditure.

Fact 1. *A customer industry's number of suppliers (from a supplier industry) increases with its intermediate input expenditures (on this supplier industry).*

Fact 1 is documented by estimating the following two equations:

$$d\ln(v_{ind,n,t}) = \beta_1 d\ln\left(pxv_{ind,n,t}\right) + \epsilon_{n,t}, \tag{1}$$

$$d\ln(v_{ind,ns,t}) = \beta_2 d\ln\left(pxv_{ind,ns,t}\right) + \epsilon_{ns,t}, \tag{2}$$

where $v_{ind,ns,t}$ ($pxv_{ind,ns,t}$) is the number of suppliers (input expenditure) that customer industry n sources from in industry s in year t, and $v_{ind,n,t}$ ($pxv_{ind,n,t}$) is customer industry n's total supplier number (input expenditure). I use the first-order difference to remove the trend in each variable so that the estimated elasticities correspond to business cycle variations.

Table 1 presents the results of regressions 1 and 2. Column (a) shows that a 1% increase in the customer industry's total intermediate input expenditure is associated with a 0.281% increase in its total number of suppliers. This positive correlation echoes the similar correlation in the case of Ford in the introduction. Column (b) further shows that this positive correlation holds between customer-supplier industry pairs. A 1% increase in a customer industry's input expenditure on a supplier industry is associated with a 0.206% increase in its number of suppliers from that supplier industry. Columns (c) and (d) use the customer industry's real sales and the supplier industry's real output price as instruments for the intermediate input expenditure, respectively. The positive correlation is still significant. This indicates that an increase in the number of suppliers is associated with an increase in either the customer industry's sales or the supplier industry's output price, which are related to the amplification and reshuffle effects illustrated in Section 5, respectively.

Table 1: Industries' Supplier Numbers and Input Expenditures

	(a) Total Supplier No.	(b) Supplier No.	(c) Supplier No.	(d) Supplier No.
$pxv_{ind,n,t}$	0.281***			
	(0.108)			
$pxv_{ind,ns,t}$		0.206***	0.332***	0.218**
		(0.0525)	(0.0748)	(0.101)
Instrumental Variable			$sales_{ind,n}$	$p_{ind,s}$
Observations	195	1,443	1,443	1,443
R^2	0.037	0.011		

Note: Data are annual from 2003 to 2016. The dependent variables are customer industries' total numbers of suppliers for column (a) and supplier numbers from each industry for columns (b)-(d). $pxv_{ind,n,t}$ is the total intermediate input expenditure of the customer industry, and $pxv_{ind,ns,t}$ is the input expenditure on each supplier industry. Columns (c) and (d) use customer industries' gross outputs ($sales_{ind,n,t}$) and supplier industries' output prices ($p_{ind,s}$) as instrumental variables for $pxv_{ind,ns,t}$, respectively. All variables are log-differenced. Input expenditures, gross outputs, and output prices are deflated by the GDP deflator. Industry and customer-supplier pair fixed effects are controlled in column (a) and columns (b)-(d), respectively. Standard errors are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

The above results show a positive correlation between the number of suppliers and input expenditures on the industry-level. It is important to show this positive correlation is consistent with firm-level evidence. Without firm-level input expenditure data, I only explore the correlation between a firm's total sales/costs (denoted by $sales_{firm,i,t}$ and $cogs_{firm,i,t}$, respectively) and its total number of suppliers (denoted by $v_{firm,i,t}$) by estimating the following two equations:

$$d\ln(v_{firm,i,t}) = \beta_3 d\ln(sales_{firm,i,t}) + \epsilon_{3,i,t}, \tag{3}$$

$$d\ln(v_{firm,i,t}) = \beta_4 d\ln(cogs_{firm,i,t}) + \epsilon_{4,i,t}. \tag{4}$$

Table 2 present results of firm-level regressions 3 and 4. A 1% increase in a firm's sales and cost of goods is associated with 0.08% and 0.06% increases in its number of suppliers, respectively. This positive correlation on the firm-level implies that firms' adjustments of supplier numbers in response to input expenditure changes are the forces behind industry-level adjustments.

The above industry and firm-level evidence shows that the extensive margin adjusts together with intermediate input expenditures (or sales) and they move in the same direction. To further compare the sizes of extensive and intensive margin adjustments, I first decompose customer industry n's

Table 2: Firms' Sales, Cost of Goods Sold, and Supplier Numbers

VARIABLES	(a) Total Supplier No.	(b) Total Supplier No.
$sales_{firm,i,t}$	0.0808***	
	(0.0130)	
$cogs_{firm,i,t}$		0.0608***
		(0.0116)
Observations	14,627	14,628
R^2	0.003	0.002

Note: Data are annual from 2003 to 2016. The dependent variables are customer firm's total number of suppliers. $sales_{firm,i,t}$ is customer firm's net sales, and $cogs_{firm,i,t}$ is its total number of suppliers. All variables are log-differenced. Sales and cost of goods sold are deflated by the GDP deflator. Firm fixed effects are controlled. Standard errors are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

input expenditure on the product of industry s as

$$pxv_{ind,ns,t} = p_{ind,s,t} x_{ind,ns,t} v_{ind,ns,t},$$

where $p_{ind,s,t}$ is the price of industry s' output and $p_{ind,s,t} x_{ind,ns,t}$ is the average input expenditure that industry n spend on a supplier in industry s. $x_{ind,ns,t}$ is therefore the average input quantity that industry n sources from a supplier in industry s.

I run the following regressions to study the responses of extensive and intensive margins to changes in customer industries' real sales:

$$dlnxv_{ind,n,t} = \beta_5 d\ln(sales_{ind,n,t}) + \epsilon_{5,n,t}, \quad (5)$$

$$dlnv_{ind,n,t} = \beta_6 d\ln(sales_{ind,n,t}) + \epsilon_{6,n,t}, \quad (6)$$

$$dlnx_{ind,n,t} = \beta_7 d\ln(sales_{ind,n,t}) + \epsilon_{7,n,t}, \quad (7)$$

where $lnv_{ind,n,t}$, $lnx_{ind,n,t}$, and $lnxv_{ind,n,t}$ are the log weighted number of suppliers, input quantity per supplier, and total input quantity. The construction of these industry-level weighted variables is described in Appendix A.3. $sales_{ind,n,t}$ is customer industry n's nominal gross output deflated by the GDP deflator.

Table 3 show the results of regressions 5 to 7. When a customer industry's real sales increases by 1%, its weighted input quantity increases by 0.831%. Meanwhile, its extensive and intensive margins increase by 0.247% and 0.580%, respectively. Thus, 30% of the input quantity adjustments are on the extensive margin.

Table 3: Customer Industries' Input Quantity, Supplier Numbers, and Sales

	(a)	(b)	(c)
VARIABLES	$dlnxv_{ind,n,t}$	$dlnv_{ind,n,t}$	$dlnx_{ind,n,t}$
$sales_{ind,n,t}$	0.831***	0.247***	0.580***
	(0.0399)	(0.0904)	(0.0975)
Observations	210	210	210
R^2	0.686	0.0347	0.145

Note: Data are annual from 2003 to 2016. The dependent variables are log-differenced customer industries' weighted input quantity, weighted supplier numbers, and weighted input quantity per supplier, respectively. $sales_{ind,n,t}$ is log-differenced customer industry n's nominal gross output deflated by the GDP deflator. Industry fixed effects are controlled. Standard errors are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

2.3 Return to More Suppliers

Fact 1 shows that customer industries adjust their number of suppliers together with intermediate input expenditures. The following fact illustrates an incentive for the extensive margin adjustment.

Fact 2. *A 1% increase in the customer industry's input supplier number is associated with a 0.035% increase in its real output.*

The productivity gain (loss) due to extensive margin adjustments is documented by estimating customer industry or firm's total factor productivities with the number of suppliers included. Estimations on both industry and firm levels are as follows:

$$d\ln q_{ind,n,t} = \beta_{81} dlnv_{ind,n,t} + \beta_{82} dlnxv_{ind,n,t} + \beta_{83} d\ln l_{ind,n,t} + \beta_{84} d\ln k_{ind,n,t} + \epsilon_{8,n,t}, \quad (8)$$

$$d\ln sales_{firm,i,t} = \beta_{91} d\ln v_{firm,i,t} + \beta_{92} d\ln xv_{firm,i,t} + \beta_{93} d\ln l_{firm,i,t} + \beta_{94} d\ln k_{firm,i,t} + \epsilon_{9,i,t}, \quad (9)$$

where $q_{ind,n,t}$ is industry n's real output at time t. $lnv_{ind,n,t}$, $lnxv_{ind,n,t}$, $l_{ind,n,t}$, and $k_{ind,n,t}$ ($lnv_{firm,i,t}$, $lnxv_{firm,i,t}$, $l_{firm,i,t}$, and $k_{firm,i,t}$) are log industry-level (firm-level) number of suppliers, intermediate input quantity, employment, and capital stock, respectively.

Table 4 presents the results of regressions 8 and 9. Industry-level regression 8 (column a) shows that controlling for capital and labor inputs, a 1% increase in intermediate input quantity is associated with a 0.34% increase in the real output of the customer industry. What is more, a 1% increase in the number of suppliers is associated with another 0.035% increase in the real output even when input quantity is already controlled for.[8] Remember that a 1% increase in intermediate

[8]There might be reserve causality problem here because a higher output incentivizes a firm to increase the number of suppliers if it leads to higher sales. This endogeneity problem may lead to an over-estimated output gain of more suppliers. However, I instrument the log-differenced number of suppliers with its one-period lag and compare the IV

Table 4: Industries' / Firms' Output and Numbers of Suppliers

	(a)	(b)	(c)
VARIABLES	Industry's Ouptut	Industry's Ouptut	Firm's Real Sales
$dlnv_{ind,n,t}$	0.035***		
	(0.0168)		
$dlnv_{ind,unweight,n,t}$		0.0166*	
		(0.009)	
$dlnxv_{ind,n,t}$	0.335***		
	(0.0244)		
$dlnxv_{ind,BEA,n,t}$		0.321***	
		(0.0213)	
$d\ln l_{ind,n,t}$	0.328***	0.292***	
	(0.0516)	(0.0499)	
$d\ln k_{ind,n,t}$	0.261*	0.202	
	(0.136)	(0.130)	
$d\ln v_{firm,i,t}$			0.0305***
			(0.008)
$d\ln xv_{firm,i,t}$			0.189***
			(0.006)
$d\ln l_{firm,i,t}$			0.439***
			(0.0137)
$d\ln k_{firm,i,t}$			0.007***
			(0.0131)
Observations	210	210	3,983
R^2	0.714	0.734	0.441

Note: Data are annual from 2003 to 2016. The dependent variables are customer industries' quantity indexes of gross output and customer firms' sales deflated by the GDP deflator, respectively. $dlnv_{ind,n,t}$, $dlnv_{ind,unweight,n,t}$, $dlnxv_{ind,n,t}$, $dlnxv_{ind,BEA,n,t}$, $d\ln l_{ind,n,t}$, and $d\ln k_{ind,n,t}$ are log-differenced customer industries' weighted supplier numbers, unweighted total supplier numbers, weighted input quantity, BEA input quantity index, number of hours, and fixed asset quantity, respectively. $d\ln v_{firm,i,t}$, $d\ln xv_{firm,i,t}$, $d\ln l_{firm,i,t}$, and $d\ln k_{firm,i,t}$ are log-differenced customer firms' total number of suppliers, real intermediate input, employment, and capital stock, respectively. Firm-level variables are deflated by their corresponding NAICS industry-level price indexes. Industry and firm fixed effects are controlled in columns (a) and (b), respectively. Standard errors are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

input quantity is associated with a 0.3% increase in the number of suppliers. Thus, a 1% increase in customer industry's intermediate input quantity is associated with an extra 0.031% output increase due to extensive margin adjustments. In Section 4, I show that the industry-level productivity gain is further amplified when industry shocks transmit in the production network and generates large aggregate fluctuations.

Column (b) replaces the weighted number of suppliers with an unweighted total number, and the weighted input quantity with the BEA intermediate input quantity index, respectively. The output gain of a 1% increase in supplier numbers becomes smaller and less significant at 0.017%,

estimates with the OLS estimates. The Hausman test cannot reject the null hypothesis that the OLS estimate of β_{81} is consistent.

which is less than half of the weighted counterpart in column (a). Comparing column (b) to column (a), I argue that ignoring the production network leads to an under-estimate of the productivity gain of more suppliers. In column (c), a similar output gain of more suppliers is found on the firm level. Controlling for the real intermediate input, employment, and capital stock, a 1% increase in customer firm's total number of suppliers is associated with a 0.03% increase in its real sales.

Table 5: Estimation of Firms' Production Function including Numbers of Suppliers

VARIABLES	(a) Olley-Pakes with exit Firm's Real Sales	(b) Olley-Pakes without exit Firm's Real Sales	(c) Levinsohn-Petrin with exit Firm's Real Sales
$\ln v_{firm,i,t}$	0.0878***	0.0878***	0.0629***
	(0.0121)	(0.0113)	(0.0105)
$\ln xv_{firm,i,t}$	0.259***	0.259***	
	(0.0137)	(0.0156)	
$\ln l_{firm,i,t}$	0.593***	0.593***	0.581***
	(0.0230)	(0.0241)	(0.0211)
$\ln k_{firm,i,t}$	0.111***	0.0471	0.212***
	(0.0418)	(0.0338)	(0.0326)

Note: Data are annual from 2003 to 2018. The dependent variables are customer firms' sales deflated by the BEA NAICS industry-level gross output price index. $\ln v_{firm,i,t}$, $\ln xv_{firm,i,t}$, $\ln l_{firm,i,t}$, and $\ln k_{firm,i,t}$ are log customer firms' total number of suppliers, real intermediate input, employment, and capital stock, respectively. Firm-level variables are deflated by their corresponding BEA NAICS industry-level price indexes. Year fixed effects and four-digit NAICS industry-level dummies are controlled. Standard errors are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

To rule out the possibility that the coefficient of supplier number is significant because it proxies firms' unobserved productivity, I estimate firms' production functions using the Olley-Pakes (Olley and Pakes, 1996) and Levinsohn-Petrin (Levinsohn and Petrin, 2003) methods. In particular, I assume that the number of suppliers is decided after observing the productivity shocks. Details of the methods I use are described in Appendix A.4.

Column (a) of Table 5 shows the results of production function estimation using Olley-Pake with firms' exit considered. The coefficient of firms' supplier numbers is larger than in the OLS estimation. A 1% increase in a firm's total number of suppliers is associated with a 0.09% increase in its real sales. Column (b) ignores firms' exits, and the coefficient value of supplier numbers does not change. In column (c), the Levinsohn-Petrin method is used with firms' exit considered. The coefficient of firms' supplier numbers is also larger than in the OLS estimation. A 1% increase in a firm's total number of suppliers is associated with a 0.06% increase in its real sales. Thus, I conclude that there is a productivity benefit of sourcing from more suppliers.

I argue that the output gain of more suppliers is most likely a productivity gain due to the return to variety in production. There are alternative motivations behind extensive margin adjustments, e.g., capacity constraint, risk control, and increasing competition among suppliers. However, in Appendix B, I argue that these incentives may exist, but are not consistent with the evidence in the data.

2.4 Extensive Margin Adjustments and Supply Chain Management Costs

In Section 2.3, I show that a larger number of suppliers increases output due to the return to variety, which is the benefit of extensive margin adjustments. In this section, I study the cost of such adjustments by estimating the following equations:

$$d3\ln(v_{ind,n,t}) = \beta_{12} d3\ln(purch\_emp_{ind,n,t}) + \epsilon_{12,n,t} \tag{10}$$

$$d3\ln(purch\_emp_{ind,n,t}) = \beta_{131} d3\ln(v_{ind,n,t}) + \beta_{132} d3\ln(pxv_{ind,n,t}) + \epsilon_{13,n,t} \tag{11}$$

where $d3.x_t = x_t - x_{t-3}$ for any variable x. $purch\_emp_{ind,n,t}$ is industry n's employment of purchasing agents.

As mentioned in Appendix A.2, the OES survey data is collected over a three-year cycle, and employment estimates for any year include employment information in the past two and a half years. Thus, I use third-order differenced variables in regressions.

Table 6: Supplier Number and Employment of Purchasing Agents

VARIABLES	(a) Total Supplier No.	(b) Purchasing Agents Emp.
$purch\_emp_{ind,n,t}$	0.272***	
	(0.0946)	
$v_{ind,n,t}$		0.220***
		(0.0684)
$pxv_{ind,n,t}$		-0.259*
		(0.143)
R^2	0.048	0.073
Observations	165	165

Note: Data are annual from 2003 to 2016. The dependent variables are an industry's number of suppliers and employment of purchasing agents, respectively. $v_{ind,n,t}$ is the industry's total number of suppliers, $pxv_{ind,n,t}$ is the total intermediate input expenditures. All variables are log and third-order differenced. Input expenditures are deflated using the GDP deflator. Industry fixed effects are controlled. Standard errors are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Fact 3. *A 1% increase in a customer industry's supply chain management cost is associated with a 0.27% increase in its number of suppliers.*

Fact 3 is a direct observation of the regression results of 10 and 11 in Table 6. Column (a) shows that a 1% increase in a customer industry's employment of purchasing agents, the proxy for supply chain management cost, is associated with a 0.27% increase in its total number of suppliers. On the other hand, column (b) shows that the management cost is negatively and insignificantly correlated with the extensive margin adjustment. Columns (a) and (b) indicate that the supply chain management cost is positively correlated with the extensive margin but not the intensive margin adjustment. These observations motivate me to assume a fixed cost of managing suppliers in the model.

3 A Model with Extensive Margin Adjustments

In this section, I present a real business model with adjustments in the number of input varieties (the extensive margin). I illustrate how the extensive margin adjustments amplify productivity shocks and the aggregate fluctuations in real GDP. In this economy, there are two types of agents, the firms (producers) and households. First, I describe the problems of producers and households. Then, I describe the timing of the model and define the equilibrium. Finally, I solve the model and illustrate how extensive margin adjustments amplify aggregate fluctuations.

3.1 Firm Producers and the Number of Input Varieties

On the production side, there is a continuum of monopolistic competitive firms with a unit measure. Each firm produces a unique and differentiated variety i. A firm produces gross output q with production labor l and management labor l_{mng} supplied by the household, and composite intermediate input X composed of inputs from other firms. The management labor l_{mng} is used by firms to manage suppliers. The output of a firm is used for consumption as well as intermediate inputs. Thus, a firm is both a customer and a supplier of intermediate inputs.

The production function of firm i is

$$q_i = ZX_i^{\alpha} l_i^{1-\alpha}, \tag{12}$$

where Z is the productivity shared by all firms in the economy. Long-run intermediate input and labor cost shares are equal to α and $1 - \alpha$, respectively.

The composite intermediate input X_i is a CES aggregator of inputs from other firms,

$$X_i = \left(\int_{m \in V_i} v_i^{\varphi} x_i(m)^{\frac{\gamma-1}{\gamma}} dm \right)^{\frac{\gamma}{\gamma-1}}, \tag{13}$$

where $x_i(m)$ is the input quantity that firm i sources from firm m, which is defined as the intensive margin. $V_i \subseteq [0, 1]$ is the set of input varieties used by firm i, which is defined as the extensive margin. Denote the number of varieties in V_i by v_i. The elasticity of substitution between different varieties is γ and $\gamma > 1$.

With the extensive margin, the productivity of a firm depends on the overall productivity, as well as how many input varieties it uses. The productivity gain of more input varieties arises when differentiated inputs in set V_i are combined. I follow Ethier (1982) to use this production technology. In particular, I use φ to control the extent of this return to input variety in production. The return to variety exists in a standard CES aggregator where the extensive margin V_i is endogenous and $\varphi = 0$. On the other hand, when $\varphi = -1/\gamma$, the return to variety is zero. I assume $\varphi > -1/\gamma$ in this paper due to the documented return to variety in Fact 2. This return to input variety in production may arise from the "exploitation of the division of labor," as argued by Ethier (1982), or from an improved quality when more input varieties are combined.

To maintain the sourcing relationship with each variety/supplier, a customer firm must employ one unit of management labor l_{mng}, the wage of which is w_{mng}. In particular, I assume management and production labor markets are segmented. This assumption is based on the observation from the OES data that purchasing agents' wages are significantly higher than the average across all industries.

Under the timing assumption 2, this management cost is a fixed cost decided before choosing intermediate input quantity. A monopolistic competitive firm i solves the following profit maximization problem

$$\pi_i = \max_{\substack{p_i, l_i, V_i \\ \{x_i(m)\}_m}} (p_i - mc_i) q_i(p_i) - w_{mng} v_i \tag{14}$$

where $q_i(p_i)$ is firm i's demand given price p_i. Under the monopolistically competitive assumption of Assumption 2, $q_i = p_i^{-\gamma} \tilde{D}_i$. $\tilde{D}_i$, is a function of aggregate consumption, intermediate inputs, and their prices, and its form under the random selection assumption of Assumption 3, $\tilde{D}$, is given in Appendix A.3. The firm sets its output price p_i. It is worth notice that the number of varieties used, v_i is determined when the set of varieties used, V_i is determined. The marginal cost of production

mc_i is derived by solving the cost minimization problem

$$\min_{\substack{l_i, V_i \\ \{x_i(m)\}_m}} \int_{m \in V_i} p_m x_i(m) dm + w l_i + w_{mng} v_i$$
$$\text{s.t.} \quad Z X_i^{\alpha} l_i^{1-\alpha} \geq q_i, \tag{15}$$

where w is the wage of production labor.

Assumption 1. *A firm takes as given the prices and quantities chosen by other firms, as well as the wages of both production and management labor in the economy.*

Under Assumption 1, given the input variety choices V_i, the marginal cost of firm i is

$$mc_i = Z^{-1} \Big(\int_{V_i} v_i^{\varphi \gamma} p_m^{1-\gamma} dm \Big)^{\frac{\alpha}{1-\gamma}} w^{1-\alpha} \cdot \alpha^{-\alpha} (1-\alpha)^{\alpha - 1}. \tag{16}$$

Equation 16 shows that firm i's marginal cost depends on not only the prices of the inputs but also the number of input varieties it uses. Using more input varieties increases the productivity and lows the marginal cost.

3.2 Households

On the household side, there is a representative household who consumes an aggregate consumption good composed of goods produced by all firms. The representative household incurs only disutility from supplying management labor, and her problem is:[9]

$$\max_{c, L_{mng}, L} \quad log\Big(C - L_{mng}^{\eta}\Big)$$
$$s.t. \quad P^C C = wL + \int_0^1 \pi_i di + w_{mng} L_{mng} \tag{17}$$
$$C = \Big(\int_0^1 c_i^{\frac{\gamma-1}{\gamma}} di \Big)^{\frac{\gamma}{\gamma-1}}$$

where the price of the aggregate consumption good is

$$P^C = \Big(\int_0^1 p_i^{1-\gamma} di \Big)^{\frac{1}{1-\gamma}} \tag{18}$$

[9] I assume that consumption and composite intermediate input goods share the same elasticity of substitution when aggregating different varieties. Under Assumption 2, γ does not matter for consumption because all varieties are used the same quantity.

I further assume total production labor supply is fixed and $L = \bar{L} = 1$. Prices are normalized so that the wage of production labor $w = 1$. Denote the Lagrange multiplier of the household's budget constraint as λ. $\lambda = \frac{\eta(\gamma-1)(1-\alpha)}{\eta(\gamma-\alpha\gamma+\alpha)-(1+\varphi\gamma)\alpha}$ is a constant, which is pinned down by $w = 1$.

Here I use a Greenwood-Hercowitz-Huffman preference for consumption and management labor. In this model, the total wage income is equal to the fixed total expenditure on consumption. As a result, households will not adjust the supply of overall management labors if I use the log and additively separable utility function. To make the extensive margin adjust, I must provide the representative household with incentives to adjust the overall management labor supply. With Greenwood-Hercowitz-Huffman utility, management labor is paid in units of the aggregate consumption. As a result, even though the consumption expenditure stays fixed, overall management labor supply fluctuates with the price of aggregate consumption.

3.3 Timing Assumption and the Equilibrium

With the problems of different agents introduced above, I describe the timing assumption and the equilibrium definition in this section.

Assumption 2. *(Timing) In each period t, there are two stages:*

Stage 1: Given expected prices, each customer firm i selects the input varieties to use V_i (thus the number of input varieties v_i). And the supply chain forms.

Stage 2: Given the supply chain, firms in V_i compete monopolistically and post prices. Customer firm i chooses input quantity $\{x_i(m)\}_m$ according to the monopolistic competitive demand schedules and produce. The household supplies labor and consumes, and goods and labor markets clear.

Assumption 2 indicates that customer firms choose input varieties before they decide input quantity used from each variety. As a result, the supply chain management cost is a fixed cost. I use a fixed cost form because in Section 2.4, I show that supply chain management costs are positively correlated with the number of input suppliers but not input expenditures. Under Assumption 2 (and Assumption 3 to avoid hold-up problems), firms' prices are constant markups over the marginal costs, i.e.,

$$p_i = \frac{\gamma}{\gamma - 1} mc_i \quad \forall\, i. \tag{19}$$

Under Assumptions 1 and 2, a competitive equilibrium can be defined as follows

Definition *1*. *A competitive equilibrium is composed of firms' output prices $\{p_i\}_i$, consumption good price P^C, and the wages of production and management labors w and w_{mng}; allocations*

$\{C, L, L_{mng}\}$, $\{x_i(m)\}_{i,m}$, *and* $\{V_i, c_i, l_i, q_i\}_i$ *such that:*

1. Given consumption good price and wages, $\{C, L, L_{mng}\}$ *solve the representative household's problem in equation 17.*

2. Given other firms' prices and the wages, $\{x_i(m)\}_{i,m}$, *and* $\{V_i, c_i, l_i, q_i\}_i$ *and* p_i *solve firm's problem in equation 15* $\forall\, i$.

3. The market of each firm's goods clears, i.e., $\forall\, i$, $q_i = c_i + \int_0^1 \mathbb{1}\{i \in V_m\} x_m(i) dm$.

4. Production labor market clears, i.e., $L = \int_0^1 l_i di$.

5. Supply chain management labor markets clear, i.e., $L_{mng} = \int_0^1 v_i di$.

In this paper, the extensive margin I focus on is the number of input varieties. Thus, I study the equilibrium where all firms are symmetric, i.e., they choose the same input quantities, number of input varieties, and labor, and post the same price. The following assumption guarantees the existence and uniqueness of such an equilibrium.

Assumption 3. *In stage 1: each customer firm i randomly selects input varieties from all firms.*

Proposition 1. *Under Assumptions 1 to 3, there is a unique symmetric equilibrium which satisfies equilibrium definition 1, and all firms behave symmetrically such that,*

1. Firms choose the same input quantity and number of varieties, price, and production and management labor, i.e., $x_i(m) = x\ \forall\, i, m \in V_i$ *and* $v_i = v,\ p_i = p,\ l_i = l\ \forall\, i$.

2. The consumption good producer uses the same quantity of goods from each firm, i.e., $c_i = c\ \forall\, i$.

Proposition 1 defines an equilibrium where firms are symmetric. From now on, any equilibrium in this section refers to this symmetric equilibrium. In this equilibrium, the extensive margin degenerates from the set of input varieties V to the number of input varieties v.

3.4 Extensive Margin Adjustments and Aggregate Fluctuations

Before I discuss the relationship between extensive margin adjustments and aggregate fluctuations, the following proposition helps understand how the extensive margin adjusts.

Proposition 2. *Under Assumptions 1 to 3,* $\forall n, s \in S$ *and in the (symmetric) equilibrium, the number of input varieties used satisfies*

$$\frac{1+\varphi\gamma}{\gamma-1} pxv = P^C \eta v^{\eta}. \tag{20}$$

Proposition 2 shows that the number of input varieties used by each firm increases with its input expenditure. This positive co-movement between input expenditure and the number of input varieties echoes the positive correlation in Fact 1. The following optimality conditions with respect to firms' input variety and quantity choices help to understand this co-movement.

$$x: \quad px = \frac{\gamma - 1}{\gamma} \alpha pq X^{-1} v^{\frac{(1+\varphi)\gamma}{\gamma-1}} x, \tag{21}$$

$$v: \quad (1+\varphi)\alpha pq X^{-1} v^{\frac{(1+\varphi)\gamma}{\gamma-1}-1} x - px = w_{mng}. \tag{22}$$

Further more, the optimality condition of the representative household with respect to the management labor supply yields

$$P^C \eta v^{\eta-1} = w_{mng}. \tag{23}$$

In equilibrium, $L_{mng} = v$. As a result, equations 21 to 23 combine to prove proposition 3.

The left and right-hand sides of equation 22 are the return and cost of input varieties, respectively. The marginal return to variety is $\frac{1+\varphi\gamma}{\gamma-1} px$, which is the increase in sales due to an additional variety given the same intermediate input expenditure. $P^C \eta v^{\eta-1}$ is the marginal (management) cost of variety. $\frac{1+\varphi\gamma}{\gamma-1}$ controls the size of the return to variety and px equals to marginal productivity of each variety. The marginal return to variety can be understood by studying equation 22. The left-hand side of equation 22 shows the marginal return to variety, which is the difference between the output gain from an additional variety and the expenditure on this variety. The expenditure on the variety is equal to the marginal productivity of a current variety according to equation 21. The output gain from an additional variety is higher than the marginal productivity of a current variety, even though they are symmetric. This difference is due to the productivity gain of more input varieties and is equal to $\frac{1+\varphi\gamma}{\gamma-1} px$.

With the above proposition showing how the extensive margin adjusts, the following proposition illustrates how it enters the aggregate output $Q = q$ and the GDP.

Proposition 3. *Under Assumptions 1 to 3 and in the (symmetric) equilibrium, the aggregate output and consumption in the economy, $Q = q$ and C can be represented as functions of only the productivity Z,*

$$Q = Z^{\frac{1}{1-\alpha-\alpha\frac{1+\varphi\gamma}{\eta(\gamma-1)}}} \cdot constant_q, \tag{24}$$

$$C = \left(1 - \alpha\frac{\gamma-1}{\gamma}\right) Z^{\frac{1}{1-\alpha-\alpha\frac{1+\varphi\gamma}{\eta(\gamma-1)}}} \cdot constant_q, \tag{25}$$

where $constant_q = \left(\left(\alpha\frac{1+\varphi\gamma}{\eta(\gamma-1)}\right)^{\alpha\frac{1+\varphi\gamma}{\eta(\gamma-1)}} \left(\frac{\gamma-1}{\gamma}\alpha\right)^{\alpha}\right)^{\frac{1}{1-\alpha-\alpha\frac{1+\varphi\gamma}{\eta(\gamma-1)}}}$.

In this economy without investment, aggregate consumption is the GDP. Equation 24 shows that without extensive margin adjustments (i.e., $\varphi = -1/\gamma$ or $\eta \to \infty$), a one percent increase in the productivity boosts aggregate output and the GDP by $\frac{1}{1-\alpha}$ percents. The industry productivity shock is amplified here through intermediate inputs. Higher productivity lowers its marginal cost not only directly but also indirectly through a lower intermediate input price, which equals the output price.

With extensive margin adjustments, the productivity shock is further amplified. This additional amplification enters the aggregate output Q as $-\alpha\frac{1+\varphi\gamma}{\eta(\gamma-1)}$ in the denominator. The size of the amplification effect is mainly determined by the ratio between the parameters governing the return to variety $\frac{1+\varphi\gamma}{\gamma-1}$ and the marginal cost of input varieties η. There is an α here because the extensive margin takes effect through the intermediate input price. Although both amplification effects appear in the denominator, the reasons behind them are different. In response to a positive productivity shock, the expansion in input varieties increases the productivity of the producer, lowers its marginal cost and price, and increases its sales. A rising sales, in turn, leads to even larger extensive margin adjustments and a further increase in productivity. This positive feedback is the reason why the amplification effect of extensive margin adjustments enters the output Q in the denominator.

Because the GDP is a constant share of the aggregate output, I conclude that the aggregate fluctuations of the GDP due to fluctuations in the productivity are amplified by extensive margin adjustments.

4 A Multi-industry Model with EMAs and Production Network

In this section, I present a multi-industry real business model with adjustments in the number of input varieties (extensive margin). I introduce multiple industries and a production network into the model for two reasons: First, I show in Sections 4.4 to 4.6 that the structure of the production network affects the amplification effect of EMAs on aggregate fluctuations; Second, the multi-industry model provides industry-level variations to match the observed data for the estimation of key parameters in Sections 5 and 6. As a result, the quantitative effects of EMAs in these two sections rely on the multi-industry framework.

In this economy, there are three types of agents, the firms (producers), the households, and the capital and consumption goods producers. First, I describe the problem of the producers, who choose the number of input varieties. Second, I introduce the problems of the households and capital and consumption goods producers. Third, I describe the timing of the model and define the equilibrium. Finally, I use the optimality condition with respect to input variety numbers to discuss the trade-off between the benefit and the cost of extensive margin adjustments.

4.1 Firm Producers and the Number of Input Varieties

On the production side, there are N industries in the economy. Each industry has a continuum of monopolistic competitive firms with a unit measure. Firm i in industry n produces a unique and differentiated variety (n, i). Denote the set of industries by $S = \{1, 2, ..., N\}$. The input-output linkages among industries form a production network.

A firm produces gross output q with capital k rented from the household, production labor l and management labor $\{l_{mng,n}\}_n$ supplied by the household, and composite intermediate input X composed of inputs from different firms. The management labor is industry-specific and labeled $l_{mng,n}$ for industry-n firms. The output of a firm is used for consumption, investment, and intermediate inputs. The production function of firm i in industry n at time t is

$$q_{n,i,t} = Z_{n,t} X_{n,i,t}^{\alpha_{x,n}} k_{n,i,t}^{\alpha_{k,n}} l_{n,i,t}^{1-\alpha_{x,n}-\alpha_{k,n}} \tag{26}$$

where $Z_{n,t}$ is the industry-specific productivity shared by all firms in industry n. Long-run capital and intermediate input cost shares are industry-specific, and equal to $\alpha_{k,n}$ and $\alpha_{x,n}$ for each industry n, respectively. Long-run labor cost share is $1 - \alpha_{k,n} - \alpha_{x,n}$.

The composite intermediate input $X_{n,i,t}$ is produced using inputs from firms in all industrys. And the extensive margin enters the model only through the composite intermediate input. The aggregation of different inputs follows a nested Dixit-Stiglitz form:

$$X_{n,i,t} = \left[\sum_{s \in S} \omega_{ns} \left(\int_{m \in V_{n,i,s,t}} v_{n,i,s,t}^{\varphi_s} x_{n,i,s,t}(m)^{\frac{\gamma_s - 1}{\gamma_s}} dm \right)^{\frac{\gamma_s(\epsilon_x - 1)}{(\gamma_s - 1)\epsilon_x}} \right]^{\frac{\epsilon_x}{\epsilon_x - 1}} \tag{27}$$

where $x_{n,i,s,t}(m)$ is the input quantity that firm i in industry n sources from firm m in industry s, or the intensive margin. $V_{n,i,s,t} \subseteq [0, 1]$ is the set of industry s varieties that firm i in industry n uses, or the extensive margin. Denote the number of varieties in $V_{n,i,s,t}$ by $v_{n,i,s,t}$. The parameter ϵ_x is the elasticity of substitution between different industries' goods in the production of composite intermediate inputs. Within each industry s, the elasticity of substitution between different varieties is γ_s and $\gamma_s > 1$. The parameter ω_{ns} indicates the importance of industry s good in the production of the composite intermediate input of firms in industry n. The parameter which governs the extent of return to variety, φ_s, is industry-specific. As in Section 3.1, I assume $\varphi_s > -1/\gamma_s$ due to the documented Fact 2. I also assume the markets of different industries' management labors are segmented because the OES data shows a large dispersion in purchasing agents' wages across industries. As a result, the wage of the management labor is industry-specific and equals $w_{mng,n,t}$ for industry n.

Denote x^* the value of any variable x when the model is evaluated at the point where the industry productivities are set equal to 1. In the estimation and simulation of Sections 5 and 6, I use

$$X_{n,i,t} = \left[\sum_{s\in S} \omega_{ns} \left(\int_{m\in V_{n,i,s,t}} \left(\frac{v_{n,i,s,t}}{v^*_{n,i,s}}\right)^{\varphi_s} \left(\frac{x_{n,i,s,t}(m)}{x^*_{n,i,s}(m)}\right)^{\frac{\gamma_s-1}{\gamma_s}} dm \right)^{\frac{\gamma_s(\epsilon_x-1)}{(\gamma_s-1)\epsilon_x}} \right]^{\frac{\epsilon_x}{\epsilon_x-1}} \tag{28}$$

so that $\{\omega_{ns}\}_{n,s}$ are the long-run shares of industry s goods in the intermediate input of industry n firms, and $\sum_{s\in S} \omega_{ns} = 1 \; \forall n \in S$.

Under the timing assumption 4, a monopolistic competitive firm i in industry n solves the following profit maximization problem

$$\pi_{n,i,t} = \max_{\substack{p_{n,i,t},k_{n,i,t},l_{n,i,t},V_{n,i,s,t}\\ \{x_{n,i,s,t}(m)\}_{s,m}}} (p_{n,i,t} - mc_{n,i,t})q_{n,i,t}(p_{n,i,t}) - w_{mng,n,t} \sum_s v_{n,i,s,t} \tag{29}$$

where $q_{n,i,t}(p_{n,i,t})$ is firm ni's demand given price $p_{n,i,t}$. Under the monopolistically competitive assumption of Assumption 4, $q_{n,i,t} = p_{n,i,t}^{-\gamma_n} \tilde{D}_{n,i,t}$, where $\tilde{D}_{n,i,t}$, is a function of aggregate consumption, intermediate inputs, and their prices, and its form under the random selection assumption of Assumption 5, $\tilde{D}_{n,t}$, is given in Appendix A.3. The firm sets its output price $p_{n,i,t}$. The marginal cost of production $mc_{n,i,t}$ is derived by solving the cost minimization problem

$$\begin{aligned} &\min_{\substack{k_{n,i,t},l_{n,i,t}\\ V_{n,i,s,t}\\ \{x_{n,i,s,t}(m)\}_{s,m}}} \sum_{s\in S} \int_{m\in V_{n,i,s,t}} p_{s,m,t} x_{n,i,s,t}(m) dm + (r_t + \delta_n) P_t^K k_{n,i,t} + w_t l_{n,i,t} + w_{mng,n,t} \sum_s v_{n,i,s,t} \\ &\text{s.t.} \quad Z_{n,t} X_{n,i,t}^{\alpha_{x,n}} k_{n,i,t}^{\alpha_{k,n}} l_{n,i,t}^{1-\alpha_{x,n}-\alpha_{k,n}} \geq q_{n,i,t}, \end{aligned} \tag{30}$$

where r_t is the net capital rental rate. δ_n is the industry-specific capital depreciation rate of industry n, and w_t is the wage of production labor. A firm in industry n rents capital at rate $r_t + \delta_n$ from the household. And the (net) capital rental rate the household charges is r_t.

Under Assumption 1, given the input variety choices $\{V_{n,i,s,t}\}_{n,i,s,t}$, the marginal cost of firm i in industry n is

$$mc_{n,i,t} = Z_{n,t}^{-1} \left[\sum_{s\in S} \omega_{ns}^{\epsilon_x} \left(\int_{V_{n,i,s,t}} v_{n,i,s,t}^{\varphi_s\gamma_s} p_{s,m,t}^{1-\gamma_s} dm \right)^{\frac{1-\epsilon_x}{1-\gamma_s}} \right]^{\frac{\alpha_{x,n}}{1-\epsilon_x}} \left((r_t+\delta_n)P_t^K\right)^{\alpha_{k,n}} w_t^{1-\alpha_{x,n}-\alpha_{k,n}} cst_{mc}, \tag{31}$$

where $cst_{mc,n} = \alpha_{x,n}^{-\alpha_{x,n}} \alpha_{k,n}^{-\alpha_{k,n}} (1 - \alpha_{x,n} - \alpha_{k,n})^{\alpha_{x,n}+\alpha_{k,n}-1}$.

4.2 Consumption and Capital Good Producers, and the Household

The above section introduces firm producers. In this section, I first describe the production of consumption and capital goods. In the economy, there are a continuum of identical and perfectly competitive consumption good producers and a continuum of identical and perfectly competitive capital good producers, both with unit measure. The cost minimization problems of the representative consumption good producer and the representative capital good producer are

$$\min_{\{c_{s,i,t}\}_{s,i}} \sum_{s \in S} \int_0^1 p_{s,i,t} c_{s,i,t} di$$

$$\text{s.t.} \quad \left[\sum_{s \in S} \xi_s^C \left(\int_0^1 c_{s,i,t}^{\frac{\gamma_s - 1}{\gamma_s}} di \right)^{\frac{\gamma_s}{\gamma_s - 1} \frac{\epsilon_c - 1}{\epsilon_c}} \right]^{\frac{\epsilon_c}{\epsilon_c - 1}} \geq C_t^P, \tag{32}$$

$$\min_{\{k_{s,i,t}\}_{s,i}} \sum_{s \in S} \int_0^1 p_{s,i,t} k_{s,i,t}^P di$$

$$\text{s.t.} \quad \left[\sum_{s \in S} \xi_s^K \left(\int_0^1 \left(k_{s,i,t}^P\right)^{\frac{\gamma_s - 1}{\gamma_s}} di \right)^{\frac{\gamma_s}{\gamma_s - 1} \frac{\epsilon_k - 1}{\epsilon_k}} \right]^{\frac{\epsilon_k}{\epsilon_k - 1}} \geq K_t^P, \tag{33}$$

where $c_{s,i,t}$ and $k_{s,i,t}^P$ are the quantity of goods that the consumption and capital producer use from firm i in industry s at t, respectively. C_t^P and K_t^P are the consumption and capital goods produced, respectively. Unlike the composite intermediate input, the consumption/capital good producer uses all input varieties $i \in [0, 1]$ in each industry. The parameter ϵ_c and ϵ_k are the elasticities of substitution between different industries' goods in the production of consumption and capital goods, respectively. The composite intermediate input, the consumption good, and the capital good have the same within-industry elasticity of substitution γ_s for inputs from industry s. The parameters ξ_s^C and ξ_s^K indicate the importances of industry s good in the production of the consumption and the capital goods, respectively. In the estimation and simulation of Sections 5 and 6, I use

$$C_t^P = \left[\sum_{s \in S} \xi_s^C \left(\int_0^1 \left(\frac{c_{s,i,t}}{c_{s,i}^*}\right)^{\frac{\gamma_s - 1}{\gamma_s}} di \right)^{\frac{\gamma_s}{\gamma_s - 1} \frac{\epsilon_c - 1}{\epsilon_c}} \right]^{\frac{\epsilon_c}{\epsilon_c - 1}},$$

$$K_t^P = \left[\sum_{s \in S} \xi_s^K \left(\int_0^1 \left(\frac{k_{s,i,t}}{k_{s,i}^*}\right)^{\frac{\gamma_s - 1}{\gamma_s}} di \right)^{\frac{\gamma_s}{\gamma_s - 1} \frac{\epsilon_k - 1}{\epsilon_k}} \right]^{\frac{\epsilon_k}{\epsilon_k - 1}}, \tag{34}$$

so that $\{\xi_s^C\}_s$ and $\{\xi_s^K\}_s$ are the long-run shares of different industries' goods in the production of the consumption and the capital goods, and $\sum_{s\in S}\xi_s^C = 1,\ \sum_{s\in S}\xi_s^K = 1$.

On the consumer side, a representative household chooses the consumption C, production labor supply L, management labor supply $\{L_{mng,n}\}_n$, and the capital stock K in the economy.

$$
\begin{aligned}
&\max_{C_t(s^t),K_{t+1}(s^t),L_t(s^t)} \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(log\left(C_t(s^t)\right)-\psi\frac{L_t(s^t)^{1+\epsilon_L}}{1+\epsilon_L}-\sum_{n\in S}L^{\eta}_{mng,n,t}(s^t)\right)\\
\text{s.t.}\quad & P_t^C(s^t)C_t(s^t)+P_t^K(s^t)\Big(K_{t+1}(s^t)-K_t(s^t)\Big)\le w_t(s^t)L_t(s^t)+r_t(s^t)P_t^K(s^t)K_t(s^t) \\
&+\sum_{n\in S}w_{mng,n,t}(s^t)L_{mng,n,t}(s^t)+\sum_{n\in S}\int_0^1\pi_{n,i,t}(s^t)di\quad \forall t \text{ and } s^t.
\end{aligned}
\tag{35}
$$

The entire economy is faced with industry productivity shocks and s^t denotes the stochastic state of the economy. Consumption and capital goods are purchased from their producers with prices P_t^C and P_t^K, and

$$
P_t^C=\left[\sum_{s\in S}(\xi_s^C)^{\epsilon_c}\left(\int_0^1 p_{s,i,t}^{1-\gamma_s}di\right)^{\frac{1-\epsilon_c}{1-\gamma_s}}\right]^{\frac{1}{1-\epsilon_c}}, \tag{36}
$$

$$
P_t^K=\left[\sum_{s\in S}(\xi_s^K)^{\epsilon_k}\left(\int_0^1 p_{s,i,t}^{1-\gamma_s}di\right)^{\frac{1-\epsilon_k}{1-\gamma_s}}\right]^{\frac{1}{1-\epsilon_k}}. \tag{37}
$$

4.3 Timing Assumption and the Equilibrium

The following assumption is a multi-industry version of Assumption 2. With this timing assumption, I define the equilibrium of this multi-industry model in this section.

Assumption 4. *(Timing) In each period t, there are two stages:*

Stage 1: Given expected prices, each customer firm i in industry n selects the input varieties to use $\{V_{n,i,s,t}\}_s$ (thus the number of input varieties $\{v_{n,i,s,t}\}_s$). And the production network forms.

Stage 2: Given the production network, firms in $V_{n,i,s}$ compete monopolistically, and post prices. Customer firm i chooses input quantity $\{x_{n,i,s}(m)\}_{s,m}$ according to the monopolistic competitive demand schedules. Households, consumption producers, and capital producers make decisions and markets clear.

Under Assumption 4 (and Assumption 5 to avoid hold-up problems), firms' prices are constant markups over the marginal costs, i.e.,

$$p_{n,i,t} = \frac{\gamma_n}{\gamma_n - 1} mc_{n,i,t}, \tag{38}$$

and the markups are industry-specific.

My paper share the same findings with Baqaee and Farhi (2019) and Pasten et al. (2019) that in frictional economies, sales are not a sufficient statistic for the industry's contribution to aggregate fluctuations as the Hulten's theorem states (Hulten, 1978). In addition to the monopolistic competition in their papers, costly extensive margin adjustments are another friction which breaks the Hulten's theorem in my model.

Under Assumptions 1 and 4, a competitive equilibrium resembling Definition 1 is presented in Appendix C.1. Similar to Section 3, I study the equilibrium where all firms within an industry are symmetric, i.e., they choose the same input quantities, number of input varieties, labor and capital inputs, and post the same price. The following assumption guarantees the existence and uniqueness of such an equilibrium.

Assumption 5. *In stage 1: each customer firm i in industry n randomly selects input varieties from an industry s, $\forall s$.*

Proposition 4. *Under Assumptions 1, 4, and 5, there is a symmetric equilibrium which satisfies equilibrium definition 1, and all firms within an industry behave symmetrically such that $\forall$ t and s^t,*

1. Firms within an industry choose the same input quantity and number of varieties, price, production and management labor, and capital input, i.e., $x_{n,i,s,t}(m)(s^t) = x_{ns,t}(s^t)\, \forall n,\ i,\ s\ \&\ m \in V_{n,i,s,t}$; and $v_{n,i,s,t}(s^t) = v_{ns,t}(s^t)\, \forall n\ \&\ i$; $p_{n,i,t}(s^t) = p_{n,t}(s^t),\ l_{n,i,t}(s^t) = l_{n,t}(s^t),\ k_{n,i,t}(s^t) = k_{n,t}(s^t)\, \forall n$ and i.

2. The consumption good producer uses the same quantity of goods from each firm in an industry n, so does the capital good producer, i.e., $c_{n,i,t}(s^t) = c_{n,t}(s^t),\ k^P_{n,i,t}(s^t) = k^P_{n,t}(s^t)\, \forall n$ and i.

Proposition 4 defines an equilibrium where firms within each industry are symmetric. From now on, any equilibrium of the multi-industry model refers to this symmetric equilibrium. In this equilibrium, the extensive margin degenerates from the set of input varieties $\{V_{ns,t}\}_{n,s,t}$ to the number of input varieties $\{v_{ns,t}\}_{n,s,t}$.

Proposition 5. *Under Assumptions 1, 4, and 5 and in the (symmetric) equilibrium, and normalize the Lagrangian multiplier of the budget constraint in household's problem 35 to be β^t in each period t. $\forall n, s \in S$, the number of input varieties used satisfies*

$$\frac{1 + \varphi_s \gamma_s}{\gamma_s - 1} p_{s,t} x_{ns,t} v_{ns,t} = \eta \Big(\sum_s v_{ns,t} \Big)^{\eta - 1} v_{ns,t}. \tag{39}$$

Similar to Proposition 3, Proposition 5 shows that the number of input varieties industry n uses from industry s increases with industry n's input expenditure on industry s, which echoes the positive correlation of industry-level evidence in Fact 1.

4.4 EMAs and Aggregate Fluctuations in A Simplified Multi-industry Model

The above sections 4.1 to 4.3 describe a full multi-sector model for estimation and computation in Sections 5 and 6. In the next sections 4.4 to 4.6, I use a simplified model to illustrate the role of the production network in the effects of extensive margin adjustments on aggregate fluctuations.

This simplified model is a multi-industry version of the "one-sector" model in Section 3. In each industry, there is only one representative firm producing a continuum of differentiated varieties with a unit measure. Varieties within an industry are assumed to be symmetric like in Proposition 4. I assume that the representative firm of industry n chooses price p_n as if it is faced with monopolistic competition. The resulting "markup" over marginal cost is $\gamma/(\gamma-1)$. Firms use intermediate inputs and labor to produce and do not use capital to produce. It follows that the simplified model is static. The industry n representative firm chooses the number of varieties v_{ns} from each industry s, as well as the input quantity x_{ns} to use from each of these varieties. v_{ns} and x_{ns} are still the extensive and intensive margins as in the full model, respectively. There are still the return to input varieties controlled by $\{\varphi_s\}_s$ and the supply chain management cost $w_{mng,n} \sum_s v_{ns}$.

I follow the Section 3 model to use a Greenwood-Hercowitz-Huffman preference here for a similar reason, i.e., to allow the overall management labor supply to fluctuate with the price of aggregate consumption even when with constant consumption expenditure. Details of the model are described in Appendix C.2.

Using the simplified model, I start the analysis of the role of production networks with the following lemma to illustrate how extensive margin adjustments affect the prices of the composite intermediate inputs, $\{\Phi_n\}_n$.

Lemma 1. *The percentage deviation of industry n's composite intermediate input price from the steady state can be represented as*

$$\hat{\Phi}_n = \sum_s \underbrace{\widetilde{\omega}_{ns}}_{Reshuffle} \hat{p}_s \underbrace{- \frac{1}{1-\epsilon_x} \frac{M_n}{1-M_n} \left(\widehat{p_n q_n} - \hat{P}^C\right)}_{Amplification} \tag{40}$$

where $\sum_s \widetilde{\omega}_{ns} = 1$ *and*

$$\widetilde{\omega}_{ns} = \frac{\omega_{ns}\left(1 - \frac{(1+\varphi_s\gamma)\left((1-\epsilon_x)-(\eta-1)M_n/B_n\right)}{(\gamma-1)+(1+\varphi_s\gamma)(1-\epsilon_x)}\right)}{1 - M_n}, \tag{41}$$

$$M_n = \sum_s \frac{\omega_{ns}}{\left(1 + \frac{\gamma-1}{(1+\varphi_s\gamma)(1-\epsilon_x)}\right)A_n} > 0, \tag{42}$$

and $B_n = \sum_s \frac{\omega_{ns}(1+\varphi_s\gamma)}{\gamma-1}$, $A_n = 1 + \frac{\eta-1}{B_n}\sum_s \frac{\omega_{ns}(1+\varphi_s\gamma)}{(\gamma-1)+(1+\varphi_s\gamma)(1-\epsilon_x)}$.

Lemma 1 shows that extensive margin adjustments have two effects on the composite intermediate input price in a production network, the amplification and the reshuffle effects. First, extensive margin adjustments amplify industry productivity shocks through the changes in sales, which can be seen in equation 40 with $0 < M_n < 1$. The intuition is as follows. A positive productivity shock to an industry transmits in the production network and lowers the marginal costs and prices of most industries. As a result, the household demands more of these industry goods, which increases their sales as well as intermediate input expenditures. These industries, in turn, use more input varieties due to a rising return to variety, which moves up along the increasing marginal (management) cost curve. Because an expansion in input varieties makes the customer industries more productive, their composite intermediate input prices decrease.

In addition to the amplification effect, extensive margin adjustments also have a reshuffle effect in the production network. Following a positive productivity shock to industry s, its price decreases, and customer industries spend less on inputs from industry s because the elasticity of substitution $\epsilon_x < 1$. As a result, customer industries use more input varieties from other industries relative to industry s. Due to the return to variety, extensive margin adjustments reduce the productivity of inputs from industry s relative to other industries. Thus, extensive margin adjustments essentially reshuffle the effects of different supplier industries' productivity shocks on the prices of customer industries. This reshuffle effect is reflected in the transformation of the intermediate input weight matrix from $\{\omega_{ns}\}_{n,s}$ into $\{\widetilde{\omega}_{ns}\}_{n,s}$.

Lemma 1 shows how extensive margin adjustments affect the intermediate input prices $\{\Phi_n\}_n$ in the production network. The following proposition further illustrates how the adjustments affect aggregate fluctuations.

Proposition 6. *The impact of industry k's productivity on the real GDP is*

$$\frac{d\ln(C)}{d\ln(Z_k)} = T(\xi^C)'\left(I - \alpha \underbrace{\widetilde{\Omega}}_{Reshuffle}\right)^{-1}\left(e_k + \alpha \underbrace{\begin{bmatrix} \widetilde{M}_1 \frac{d\ln(p_1q_1)}{d\ln(Z_k)} \\ \widetilde{M}_2 \frac{d\ln(p_2q_2)}{d\ln(Z_k)} \\ \vdots \\ \widetilde{M}_N \frac{d\ln(p_Nq_N)}{d\ln(Z_k)} \end{bmatrix}}_{Amplification}\right), \tag{43}$$

where $\xi^C = [\xi^C_1, \xi^C_2, ..., \xi^C_N]'$, $\widetilde{M}_n = \frac{1}{1-\epsilon}\frac{M_n}{1-M_n}\, \forall n$, $\widetilde{\Omega} = \{\widetilde{\omega}_{ns}\}_{ns}$, $T = \left(I - \alpha(\xi^C)'\left(I-\alpha\widetilde{\Omega}\right)^{-1}\left[\widetilde{M}_1, \widetilde{M}_2, ..., \widetilde{M}_N\right]'\right)^{-1}$, *and* e_k *is a* $N \times 1$ *column vector with k-th entry equal to 1 and other entries equal to 0. I is an identity matrix of dimension* $N \times N$.

Proposition 6 shows that extensive margin adjustments affect real GDP also through the amplification and the reshuffle effects. The amplification effect appears as $\alpha \sum_l \widetilde{M}_n \frac{d\ln(p_nq_n)}{d\ln(Z_k)}$, and is positive for most industries n. A positive productivity shock to industry k increases the sales of most industries n, and thus positive $\frac{d\ln(p_nq_n)}{d\ln(Z_k)}$. Given that $\widetilde{M}_n > 0$, extensive margin adjustments add to the rise in real GDP in response to the positive shock. As a result, with extensive margin adjustments, industry productivity shocks generate larger aggregate fluctuations than without the adjustments.

The reshuffle effect, on the other hand, works through the matrix $\left(I - \alpha\widetilde{\Omega}\right)^{-1}$, which is a generalized version of the Leontief inverse mentioned in Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012). This generalization allows for monopolistic competition, non-unit elasticity of substitution, and extensive margin adjustments. It captures how an industry productivity shock affects the real GDP directly by changing the price of the shocked industry and indirectly through the propagation to other industries. Notice that when the return to variety in an industry s is large (i.e., large φ_s) compared to other industries, there are larger extensive margin adjustments in this industry than in others. Then, $\widetilde{\omega}_{ns} > \omega_{ns}$ and $\sum_{s' \neq s} \widetilde{\omega}_{ns'} < \sum_{s' \neq s} \omega_{ns'}$. This indicates that the effect of the productivity shock to industry s is reshuffled to other supplier industries by extensive margin adjustments.

When the disutility of management labor supply is infinitely large, i.e., $\eta \to \infty$, or the return to variety is zero ($\varphi = -1/\gamma$), $\widetilde{\Omega} = \Omega$ and $\widetilde{M}_n = 0\ \forall\ n$. In this case, both the amplification and reshuffle effects disappear.

4.5 A Simple Illustration of the Role of Production Network

In the above section, I discuss the mechanism of the amplification effect. In this section, I further use a simple example to illustrate how extensive margin adjustments and the amplification effect depend on the structure of the production network. To illustrate the role of the production network, I compare a horizontal economy with a vertical one.

(a) Horizontal **(b)** Vertical

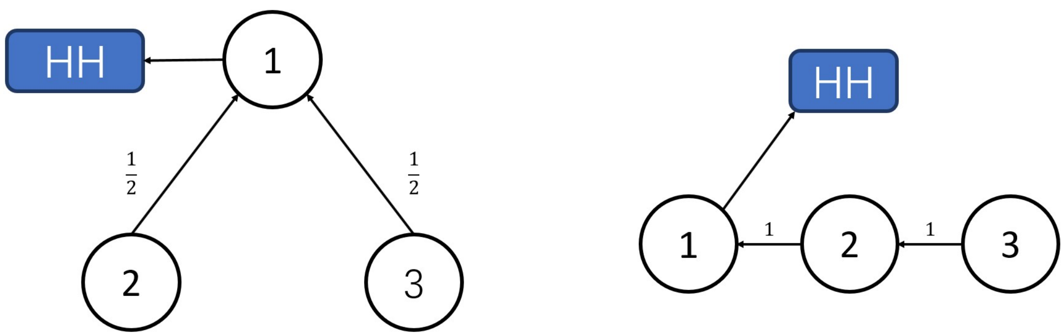

Figure 2: Horizontal vs. Vertical Economies

Note: Panels (a) and (b) plot a horizontal and a vertical economy, respectively. In both economies, the household (HH) consumes only industry 1's goods. In the horizontal economy, industry 1 uses intermediate inputs from both industries 2 and 3 with equal weights. Industries 2 and 3 use labor to produce. In the vertical economy, industry 1 uses inputs only from industry 2. And industry 2 uses inputs only from industry 3. Industry 3 uses labor to produce.

Figure 2 plots a horizontal (panel 2a) and a vertical economy (panel 2b). In both economies, the representative household (HH) only consumes industry 1's goods. I still assume that wage $w = 1$, and labor supply is fixed at 1. Then, the $P^C C$ is still a constant.

In the horizontal economy, industry 1 sources half of its intermediate inputs from industry 2 and the other half from industry 3. Industries 2 and 3 use labor to produce. In the vertical economy, industry 1 sources all of its intermediate inputs from industry 2, and industry 2 sources all of its intermediate inputs from industry 3. Industry 3 uses labor to produce.

$$d \ln C = \frac{1}{2} \frac{1}{1-\frac{1}{\eta(\gamma-1)}} \left(d \ln Z_2 + d \ln Z_3 \right) \qquad Horizontal \qquad (44)$$

$$d \ln C = \frac{1}{1-\frac{2}{\eta(\gamma-1)}} \left(d \ln Z_2 + d \ln Z_3 \right) \qquad Vertical \qquad (45)$$

Equations 44 and 45 show the responses of real GDP to industry 2 and 3's productivity shocks Z_2 and Z_3 in the horizontal and vertical economies, respectively. The amplification effects of extensive margin adjustments enter the responses of real GDP as $\frac{1}{\eta(\gamma-1)}$ and $\frac{2}{\eta(\gamma-1)}$ in the denominator, respectively. The amplification effect is larger in the vertical economy than in the horizontal one due to the following reason: In the vertical economy, extensive margin exists in two linkages, one between industries 1 and 2, and the other between industries 2 and 3; and extensive margin adjustments in the two linkages are complementary. When industry 1 uses more variety from industry 2, its productivity increases, and thus the sales. As a result, industry 2's sales increases due to a higher input demand from industry 1. In turn, industry 2 sources from more varieties in industry 3 and becomes more productive. This productivity gain of industry 2 due to more input varieties further lowers the cost and price of industry 1 and boosts the sales of industry 1. As a result, industry 1 uses even more varieties from industry 2. This complementarity between extensive margin adjustments on different linkages along the same supply chain reinforces each other. Consequently, the longer a supply chain is, the larger the amplification effect of extensive margin adjustments is.[10] To be more precise, the amplification effect depends on the lengths of supply chains adjusted by the density of the linkages. Industries can be highly connected. However, if the intermediate input flows between some industries are extremely small, those linkages are as if broken. It follows that the amplification effects along supply chains with less dense linkages are smaller than those with the same lengths but denser linkages. With that said, the production network structure is important for the amplification effect of extensive margin adjustments.

What is also worth notice is that in the vertical economy above, the production network is not amplifying productivity shocks. A 1% increase in industry 3's productivity leads to the same 1% increase in GDP. Without extensive margin adjustments, industry productivity shocks propagate in the production network by changing the marginal costs and prices of connected industries. In a vertical economy where only the most downstream industry 1 supplies consumption goods, a shock to industry 3 affects the marginal cost of industry 1 and thus the price of consumption by the same size of the shock itself. In contrast, extensive margin adjustments affect the productivity and marginal costs of industries along the supply chain. Even if the production network does not amplify industry productivity shocks directly, it affects the extensive margin adjustments triggered by productivity shocks. As a result, the production network structure still affects aggregate fluctuations indirectly through extensive margin adjustments. From another angle, industry productivity shocks propagate in the network from upstream to downstream while the effect of extensive margin adjustments propagates in the opposite direction, i.e., from downstream to upstream. Thus, the production network propagates industry productivity shocks and affects extensive margin adjustments in different ways, although in both ways, it amplifies aggregate fluctuations.

[10] When industries are defined infinitely fine, the supply chain becomes infinitely long. However, the smaller the industry is, the fewer input varieties it has, and the less flexible it can adjust the number of input varieties. Thus, the amplification effect of extensive margin adjustment is still finite.

4.6 Back-of-the-envelope Calculation

Sections 4.4 to 4.5 show how extensive margin adjustments affect the transmission of industry productivity shocks in the production network and the aggregate fluctuations. The sizes of these aggregate effects depend on the sizes of extensive margin adjustments and the return to variety and the production network structure.

In this section, I use some back-of-the-envelope calculation results to illustrate the roles of different parameters (including the network structure) in the aggregate effects of extensive margin adjustments. Compared to the quantitative analysis of the full model in Sections 5 and 6, this simple calculation allows me to explicitly separate the amplification effect and the reshuffle effect. It also allows for a wider range of values for the parameter ϵ_x because some combinations of parameter values lead to violation of the Blanchard-Kuhn conditions when simulating the full model with capital. I also set $\epsilon_c = 1$ to focus on the substitution in the production network, which is not far from the 0.8 in the full model. I rely on the parameter values calibrated and estimated in the baseline model of Sections 5 and 6. In particular, I use the medians of calibrated γ_s and α_s there as the values of γ and α in the simplified model, respectively. Then, the labor income share of an industry in the simplified model is the sum of the labor and capital income shares in the full model. In the calibration of the full model, the heterogeneity in industry-level returns to variety is purely determined by the heterogeneity in industry markups. To maintain the heterogeneity in the returns to variety in the simplified model in which industries share the same markup, I artificially adjust the values of $\{\varphi_s\}_s$ so that the industry-level returns to variety $\frac{1+\varphi_s\gamma}{\gamma-1}$ are the same as those in the full model. I use the input weight matrix calibrated to the data as in the full model. I also simulate industry productivities using the persistencies of industry productivities and the covariance matrix of productivity shocks calibrated in the full model. In the back-of-the-envelope calculation, I ignore the second-order effects of extensive margin adjustments on prices.

Table 7 compares the real GDP standard deviations under different settings or parameter values. Columns (a) to (d) show the baseline model, the models with a higher return to variety, a higher elasticity of substitution across industries, and an artificial network, respectively. In the artificial network, I use the order of Agriculture, forestry, fishing, and hunting, Mining, Utilities, Construction, Manufacturing, Wholesale trade, Retail trade, Transportation and warehousing, Information, Finance and insurance, real estate, rental, and leasing, Professional and business services, Educational services, Health care and social assistance, Arts, entertainment, recreation, accommodation, and food services, and Other services, except government, and Government, where the former of any two adjacent industries spend 99% of its input expenditures on the latter industry. Government spends 99% of its input expenditures on Agriculture. Other entries in the input weight matrix are adjusted such that the input weight matrix of the artificial network, $\{\omega^{vert}_{ns'}\}_{ns}$ satisfy

Table 7: Standard deviation of real GDP under different model settings and parameter values

	(a) Baseline	(b) Return to variety ↑	(c) Substitution ↑	(d) Artificial network
Parameters				
Elasticity of substitution	0.175	0.175	2.0	0.175
Median return to variety	0.224	0.298	0.224	0.240
Results				
With EMA	1.077%	1.109%	1.081%	1.044%
Without EMA	0.991%	0.991%	0.991%	0.971%
With EMA / without EMA	1.087	1.119	1.090	1.076
Reshuffle effect only	0.992%	0.993%	0.992%	0.971%

Note: The table shows the standard deviation of log real GDP. Baseline parameter values are based on the calibration and estimation of the baseline full model in Sections 5 and 6. Median return to variety is the median of $\left\{\frac{1+\varphi_s\gamma}{\gamma-1}\right\}_s$. "*With EMA*" is the model with extensive margin adjustments; "*Without EMA*" is the model in which the extensive margin is fixed in simulation. "*With EMA / without EMA*" is the ratio between the real GDP standard deviations of the models with and without extensive margin adjustments. "*Reshuffle effect only*" is the model without the amplification effect, i.e., $\widetilde{M}_n = 0\ \forall\, n$. From columns (a) to (d), I compare the baseline model with models with a higher return to variety, a higher elasticity of substitution across industries, and an artificial economy in which a downstream industry concentrates 99% of its input expenditure on one upstream industry, respectively.

$\sum_{\omega_{ns}^{vert}\neq 0.99}\omega_{ns'}^{vert} = 1\ \forall\, n$ and $\omega_{ns'}^{vert}/\omega_{ns''}^{vert} = \omega_{ns'}/\omega_{ns''}\ \forall\, n, s', s''\ s.t.\ \omega_{ns'}^{vert} \neq 0.99\ and\ \omega_{ns''}^{vert} \neq 0.99$. For each of these settings, I compare the real GDP standard deviations of models with extensive margin adjustments, with the extensive margin fixed, and in which the amplification effect is set to zero.

In the baseline setting (column a), the real GDP standard deviation is 1.077% with extensive margin adjustments. Compared to the 1% without extensive margin adjustments, aggregate fluctuations are amplified by the adjustments. If I mute the amplification effect by setting $\widetilde{M}_n = 0\ \forall\, n$, the real GDP standard deviation is 1% and close to that without extensive margin adjustments. Indeed, this is true for all settings from columns (a) to (d). Thus, I conclude that the reshuffle effect is negligible for the aggregate, although it matters for co-movements between industries' outputs. In column (b), I increase the median return to variety $\left(\frac{1+\varphi_s\gamma}{\gamma-1}\right)$ from 0.224 to 0.298. The standard deviation without extensive margin adjustments remains the same because the extensive margin does not move around. In contrast, the standard deviation of real GDP with extensive margin adjustments increases to 1.11%. This is because a higher return to variety not only incentivizes larger extensive margin adjustments but also generates more productivity gain or loss. As a result, aggregate fluctuations increase with the return to variety. On the other hand, if the elasticity of substitution across industries increases from 0.175 to 2 as in column (c), the reshuffle effect does not change. The amplification effect is slightly larger, resulting in a standard deviation of 1.081%. The reason is that the elasticity of substitution matters for the co-movements between the outputs of different industries, but most of the output movements of individual industries are offset by each other when aggregated to the real GDP. Lastly, in an artificial network (column d), the transmission of industry shocks changes, and the standard deviation of real GDP without extensive margin adjustments becomes 0.97%. More importantly, the size of the amplification effect of extensive margin adjustments also changes. The real GDP standard deviation with extensive margin adjustments is now only 1.076 times that without the adjustments.

5 Estimation and Simulation Results

In this section, I estimate and simulate the full multi-industry model in Section 4. First, I introduce the identification strategy of the key parameters. Second, I display the impulse responses of real GDP and industry-level outputs and prices to a Mining's productivity shock to illustrate how extensive margin adjustments affect shock transmission and aggregate fluctuations. I also show that EMA amplifies aggregate fluctuations: The real GDP standard deviation is 1.88% with EMA and 1.51% without EMA. Third, I show that the return to variety and the curvature of the management cost function significantly affect the amplification by EMA. Fourth, I show that the amplification effect of EMA is smaller in production networks with shorter (weighted) supply chains.

I estimate the model according to the indirect inference algorithm introduced in Appendix D. In the baseline model with extensive margin adjustments, the estimated intermediate input elasticity of substitution between industries ϵ_x is 0.175. The curvature of the supply chain management cost function (i.e., the disutility function of management labor) η is estimated to be 3.71. I also calibrate $AR(1)$ industry productivity processes during the estimation. I use the second-order perturbation to simulate the full model with calibrated and estimated parameters (including the industry productivity processes). The reason for second-order perturbation is to improve the approximation accuracy because the non-linearity generated by a small elasticity of substitution is non-negligible. In particular, I pruned the approximation to avoid explosive sample paths following Kim et al. (2008).[11] In some of the alternative model settings, I re-estimate the model before simulation.

5.1 Identification Strategy of Key Parameters

The key parameters to identify in this model are those governing the return to variety, the curvature of the supply chain management cost function, and the input elasticity of substitution across

[11] For a review of the perturbation and pruning methods, see Fernández-Villaverde et al. (2016).

industries. In particular, I estimate key parameters using indirect inference following Gourieroux et al. (1993) and Smith Jr. (1993). The detailed indirect inference method is described in Appendix D. The calibration of other parameters is described in Appendix E.

In the simplified model of Section 4, I find extensive margin adjustments amplify the aggregate fluctuation by almost 50%, and the amplification effect is mainly determined by the ratio between the parameters governing the return to variety $\frac{1+\varphi_s\gamma_s}{\gamma_s-1}$ and the curvature of the supply chain management cost η. The back-of-the-envelope calculation shows that the input elasticity of substitution across industries ϵ_x also matters for the amplification effect. As a result, these three key parameters are the focus of the identification strategy.

When input expenditures increase, return to variety increases, and firms pay higher management costs for more input varieties. As a result, I rely on the responses of input supplier numbers to changes in input expenditures to identify the curvature of the supply chain management cost η, (i.e., the curvature of the disutility function of management labor). To implement it, sum up both sides of equation 39 by supplier industry s and take log-difference , we have

$$d\ln\Big(\sum_s \frac{\frac{1+\varphi_s\gamma_s}{\gamma_s-1}}{\sum_{s'}\frac{1+\varphi_{s'}\gamma_{s'}}{\gamma_{s'}-1}} p_{s,t}x_{ns,t}v_{ns,t}\Big) = \eta d\ln\big(\sum_s v_{ns,t}\big) \tag{46}$$

Equation 46 states that in response to a 1% increase in the weighted input expenditures, the total number of input varieties of the customer industry increases by $\frac{1}{\eta}$%. Ideally, the coefficient of the regression of $d\ln\big(\sum_s v_{ns,t}\big)$ on $d\ln\Big(\sum_s\big(\frac{1+\varphi_s\gamma_s}{\gamma_s-1}/\sum_{s'}\frac{1+\varphi_{s'}\gamma_{s'}}{\gamma_{s'}-1}p_{s,t}x_{ns,t}v_{ns,t}\big)\Big)$ identifies η. However, the weights governed by the return to variety, $\{\frac{1+\varphi_s\gamma_s}{\gamma_s-1}\}_s$ are among the key parameters to be identified. Thus, the identification of η depends on the identification of the return to varieties. As a result, I instead regress $d\ln\big(\sum_s v_{ns,t}\big)$ on $d\ln\Big(\sum_s p_{s,t}x_{ns,t}v_{ns,t}\Big)$ to identify η in indirect inference estimation. I can do this because indirect inference estimation tolerates the mis-specification of the regressions in the auxiliary model, the details of which are introduced in Appendix D.

On the other hand, the identification of the return to variety, $\{\frac{1+\varphi_s\gamma_s}{\gamma_s-1}\}_s$ also depends on η. The intuition behind the identification is that a larger return to variety relative to the marginal management cost incentivizes firms to use more varieties. In turn, firms pay higher supply chain management costs and the share of the management costs in the sales is larger. To see this, sum up both sides of equation 39 by supplier industry s and then by customer industry n, divide by total sales in the economy, reorganize, and focus on the steady state, we have:

$$\sum_n \left(\frac{p_n^* q_n^*}{\sum_{n'} p_{n'}^* q_{n'}^*} \frac{\alpha_x(\gamma_n - 1)}{\gamma_n} \sum_s \omega_{ns} \frac{1 + \varphi_s \gamma_s}{(\gamma_s - 1)\eta} \right) = \frac{\sum_n \left(\sum_s v_{ns,t} \right)^\eta}{\sum_n p_n^* q_n^*} \tag{47}$$

The left-hand side of equation 47 is the ratio between the return to variety $\frac{1+\varphi_s\gamma_s}{\gamma_s - 1}$ and management cost curvature η weighted by intermediate input weight ω_{ns}, which is further multiplied by the steady-state share of each industry's input expenditures in the economy-wise total sales. The right-hand side is the steady-state share of economy-wise supply chain management costs in total sales, which is observed in the data. To be more specific, APQC[12] surveys around 2,000 firms in America about the share of supply chain management costs in their revenues. The average share of the costs directly related to managing input suppliers in firms' revenues is about 3%. I rely on this number to identify the return to variety. Lacking information on industry-specific return to variety, I assume $\varphi_s \gamma_s = \overline{\varphi\gamma}$. I choose the value of $\overline{\varphi\gamma}$. Then $\varphi_s = 1/\gamma_s \; \forall \; s$ and the industry-level returns to variety are determined by industry-level markups.

The identification of the intermediate input elasticity of substitution across industries follows Atalay (2017). The response of intermediate input shares to changes in supplier industries' relative price identifies the elasticity. To implement it, I rely to the following equation constructed from the optimality condition with respect to the input quantity,

$$\begin{aligned} d\ln\left(\frac{p_{s,t} x_{ns,t} v_{ns,t}}{\sum_{s'} p_{s',t} x_{ns',t} v_{ns',t}} \right) &= (1 - \epsilon_x) d\ln(p_{s,t}) - (1 - \epsilon_x) d\ln(\Phi_{n,t}) \\ &+ \frac{(1 - \varphi_s \gamma_s)(\epsilon_x - 1)}{(\gamma_s - 1)} d\ln(v_{ns,t}). \end{aligned} \tag{48}$$

The extensive margin leads to two deviations from Atalay (2017). First, the composite intermediate input price $\Phi_{n,t}$ includes the extensive margin. Second, the extensive margin $d\ln(v_{n,s,t})$ affects the supplier industry's effective productivity as an input, and thus the share of this industry s goods in the intermediate inputs of the customer industry n. Due to these two deviations, the identification of ϵ_x is also affected by the other two key parameters which determine extensive margin adjustments.

5.2 Extensive Margin Adjustments and Shock Transmission

Figure 3 plots the impulse response function of the real GDP in response to a 1% positive productivity shock to the Mining industry in period $t = 1$. Productivities of other industries in all periods

[12] www.apqc.org

are set to their steady-state values. In the baseline model with extensive margin adjustments, real GDP increases by 0.27% in period $t = 1$. Without extensive margin adjustments, real GDP still increases, but only by 0.2%.

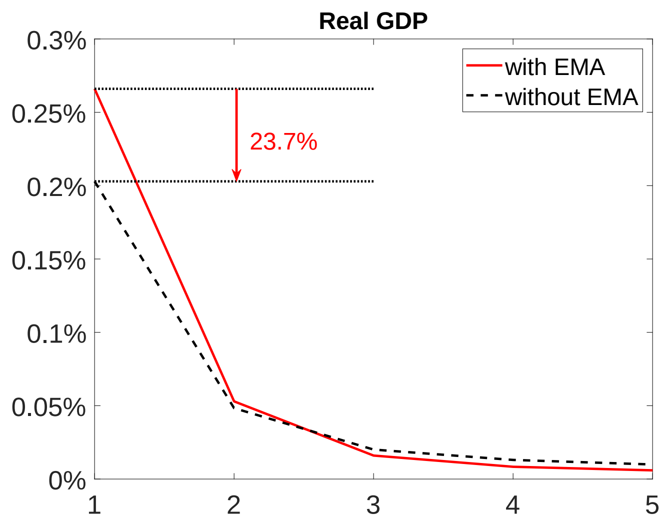

Figure 3: IRFs of real GDP to a positive productivity shock to Mining

Notes: Impulse responses are deviations of the real GDP from the steady state in response to a 1% positive Mining's productivity shock. Productivities of other industries in all periods are kept at their steady-state values. "$With\ EMA$" is the simulation result with baseline parameter values and extensive margin adjustments. "$Without\ EMA$" is the simulation result with baseline parameter and steady-state values, but without extensive margin adjustments. The calculation uses the supply chain management cost data from APQC.

Figures 4 and 5 further show how extensive margin adjustments amplify industry productivity shocks. Following a 1% positive productivity shock to the Mining industry in period $t = 1$, Manufacturing's marginal cost increases, and thus its price decreases as in the left panel of Figure 4. As a result, Manufacturing's sales and thus output increase as in the right panel of Figure 4. Comparing the two lines with and without extensive margin adjustments, we can see that the adjustments add to the decrease in Manufacturing's price and the increase in its output. Due to the rising sales following the positive shock, Manufacturing firms purchase more intermediate inputs. A larger intermediate input expenditure raises the return to input variety, which incentivizes firms to use more varieties. This expansion of input varieties, in turn, boosts the productivity of manufacturing firms, resulting in an extra decrease in price and an extra increase in output. This additional increase in output due to extensive margin adjustments is the amplification effect.

On the other hand, Figure 6 shows the reshuffle effect of extensive margin adjustments. Following a 1% positive productivity shock to the Mining industry in period $t = 1$, Mining firms see a decline in their prices. Because the input elasticity of substitution ϵ_x is smaller than one, Manufacturing firms increase the expenditures on inputs from the Utilities industry relative to the

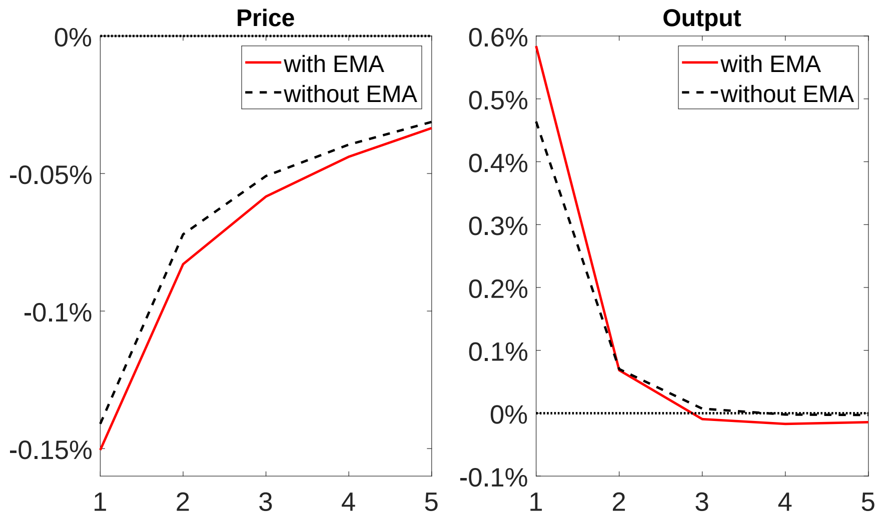

Figure 4: IRFs of Manufacturing's price and output to a positive productivity shock to Mining

Notes: Impulse responses are deviations of Manufacturing's price (left panel) and output (right panel) from the steady state in response to a 1% positive Mining's productivity shock. Productivities of other industries in all periods are kept at their steady-state values. "$With\ EMA$" is the simulation result with baseline parameter values and extensive margin adjustments. "$Without\ EMA$" is the simulation result with baseline parameter and steady-state values, but without extensive margin adjustments. The calculation uses the supply chain management cost data from APQC.

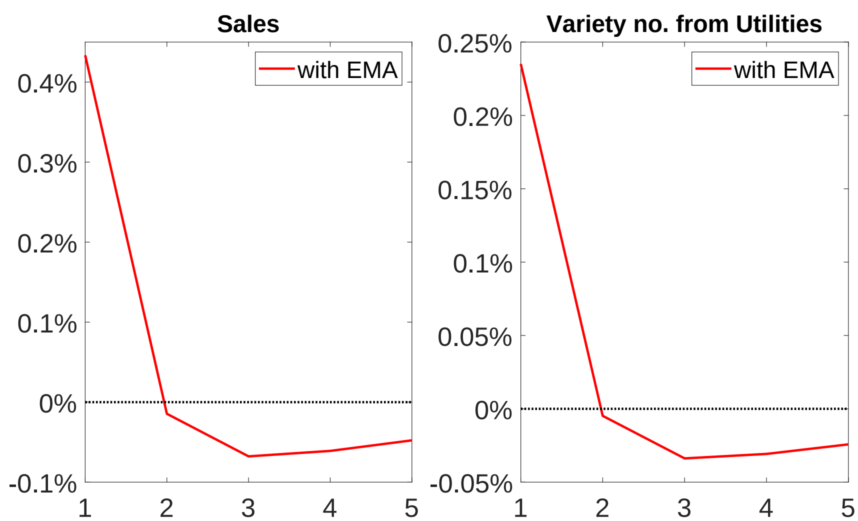

Figure 5: IRFs of Manufacturing's sales and variety no. to a positive productivity shock to Mining

Notes: Impulse responses are deviations of Manufacturing's sales (left panel) and its number of input varieties in the Utilities industry (right panel) from the steady state in response to a 1% positive Mining's productivity shock. Productivities of other industries in all periods are kept at their steady-state values. "$With\ EMA$" is the simulation result with baseline parameter values and extensive margin adjustments. The calculation uses the supply chain management cost data from APQC.

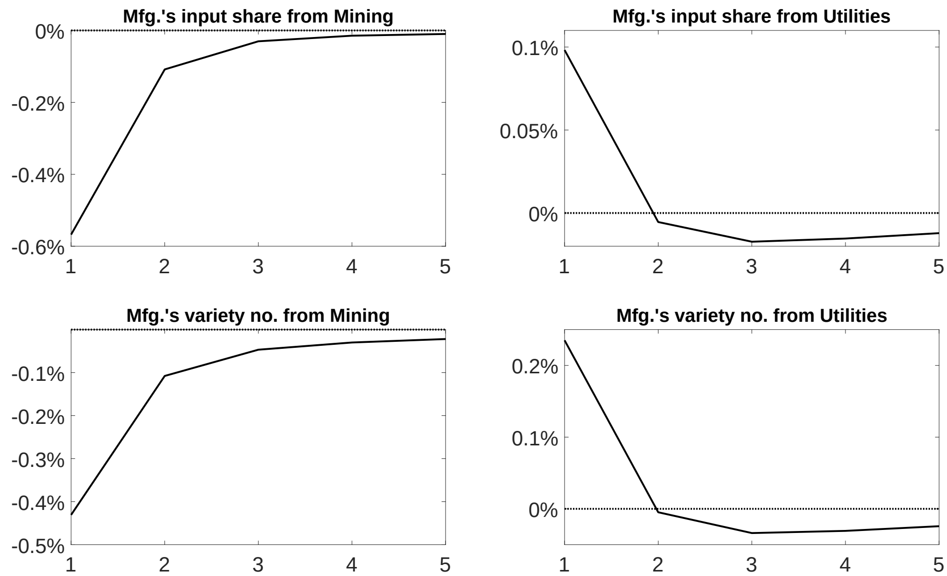

Figure 6: IRFs of Manufacturing's input shares and variety no. to a positive productivity shock to Mining

Notes: Impulse responses are deviations of Manufacturing's input shares and the number of input varieties (in Mining and Utilities) from the steady state in response to a 1% positive Mining's productivity shock. The extensive margin is allowed to adjust. Productivities of other industries in all periods are kept at their steady-state values. The calculation uses the supply chain management cost data from APQC.

Mining industry. This shift of intermediate input shares from Mining to Utilities is illustrated by the top two graphs in Figure 6. It follows that the return to Utilities varieties increases relative to the return to Mining varieties. As a result, Manufacturing firms use more Utilities varieties and fewer Mining varieties, which is shown in the bottom two plots in Figure 6. This means that extensive margin adjustments make Mining inputs less productive while Utilities inputs more productive. In other words, the adjustments reshuffle the positive Mining's productivity shock to the Utilities industry.

5.3 Extensive Margin Adjustments and Aggregate Fluctuations

The above impulse responses illustrate how extensive margin adjustments affect the transmission of industry productivity shocks and real GDP. In this section, I study the aggregate fluctuations of real GDP.

Table 8 compares the real GDP standard deviations under various settings.[13] In the baseline

[13]For both Tables 8 and 10, I simulate the model for 5,000 periods and use the real GDP series from periods 2,501 to 4,500. I calculate the standard deviation of hp-filtered real GDP to avoid medium- frequency fluctuations.

Table 8: Real GDP standard deviations under different settings

	(a) Baseline	(b) Inelastic Labor	(c) Artificial network
$With\ EMA$	1.88%		2.12%
$Without\ EMA$	1.51%	0.86%	1.58%
$Without\ \ EMA\ /\ with\ EMA$	0.81	0.46	0.74

Note: Real GDP is GDP deflated by the GDP deflator. Standard deviation is that of hp-filtered log real GDP. Column (a) shows simulation results using the parameter values calibrated and estimated as in Section 5.1. Column (b) is based on column (a) and simulated with inelastic production labor supply (Frisch elasticity of labor supply equal to 10^{-6}). Column (c) uses an artificial production network structure as described in Section 4.4 in simulation. "$With\ EMA$" is the model with extensive margin adjustments; "$Without\ EMA$" is the model simulated without extensive margin adjustments. "$Without\ EMA\ /\ with\ EMA$" is the ratio between the real GDP standard deviations of the models without and with extensive margin adjustments for columns (a) and (c). In column (b), "$Without\ EMA\ /\ with\ EMA$" is the ratio between the real GDP standard deviations of the model with inelastic labor supply and without extensive margin adjustments and the baseline model with extensive margin adjustments. Calculation uses the supply chain management cost data from APQC.

model of column (a), the input elasticity of substitution between different industries and the curvature of the supply chain management cost are estimated to be 0.175 and 3.71, respectively. With extensive margin adjustments, the standard deviation of real GDP is 1.88%. If the number of input varieties is kept at their steady-state values, the real GDP standard deviation is only 0.81 of that with extensive margin adjustments. Thus, extensive margin adjustments amplify aggregate fluctuations.

To get a sense of how large the amplification is, I compare the baseline model to a model with an inelastic supply of production labor. Column (b) uses the same industry productivities, and parameter values as in column (a) expect that the production labor supply is almost inelastic ($\epsilon_L = 10^7$). Business cycle accounting says the fluctuations in labor input explain a large share of aggregate fluctuations in real GDP. Thus as expected, fixing (production) labor supply reduces the real GDP standard deviation to only 0.46 of that in the baseline model with extensive margin adjustments. As a result, extensive margin adjustments account for 0.19 of the baseline aggregate fluctuation, which is about 54% of that due to fluctuations in (production) labor supply.

Column (c) in Table 8 shows that the production network structure matters for the size of the amplification. In an artificial production network, aggregate fluctuations become larger in both models with and without extensive margin adjustments.[14] More importantly, the real GDP standard deviation without extensive margin adjustments is now 0.74 of that with the adjustments, which is smaller than the 0.81 in the baseline model. The production network structure affects not only the

[14] $\{\varphi_s\}_s$ are recalibrated to keep the steady-state supply chain management cost share at the baseline value.

transmission of industry shocks but also how extensive margin adjustments on different linkages complement and reinforce each other.

5.4 Return to Variety, Management Cost, and Aggregate Fluctuations

Table 9: Real GDP standard deviations and return to variety

Management cost share in total revenue	(a) 1.5%	(b) 3%	(c) 6%
Parameters			
Median return to variety	0.112	0.224	0.446
Results			
Real GDP with EMA	1.67%	1.88%	2.59%
Real GDP w/o EMA	1.51%	1.51%	1.51%
With EMA / without EMA	1.11	1.24	1.44

Note: Real GDP is GDP deflated by the GDP deflator. Standard deviation is that of hp-filtered log real GDP. Column (b) shows simulation results using the baseline parameter values calibrated and estimated as in Section 5.1, in which the economy-wise share of supply chain management costs in total sales is 3%. Columns (a) and (c) are the results using the baseline parameter values except that the returns to variety are calibrated such that the economy-wise share of supply chain management costs in total sales are 1.5% and 6%, respectively. The median return to variety is the median of $\{\frac{1+\varphi_s\gamma_s}{\gamma_s-1}\}_s$. "*With EMA*" is the model with extensive margin adjustments; "*Without EMA*" is the model simulated without extensive margin adjustments. "*With EMA / without EMA*" is the ratio between the real GDP standard deviations of the models with and without extensive margin adjustments. Calculation uses the supply chain management cost data from APQC.

Table 9 compares the standard deviations of real GDP under different returns to variety. By changing the supply chain management cost share (in total revenue) from the baseline 3% to 1.5% and 6%, the median returns to variety vary from 0.112 to 0.446.[15] When the return to variety increases to 0.446, firms adjust their number of varieties by the same percentages as in the baseline model. The same adjustments of input varieties, however, generate larger movements in productivity with a larger return to variety. As a result, real GDP standard deviation increases from 1.88% to 2.59%, which is 1.44 times that without extensive margin adjustments. In comparison, when the management cost share decreases to 1.5%, extensive margin adjustments increase aggregate fluctuations by a smaller amount of 1.11 times that without the adjustments.

On the other hand, Table 10 shows how the cost of managing input varieties affects aggregate fluctuations. Keeping the return to variety at its baseline value, when the curvature of supply chain management cost function (i.e., the curvature of the disutility of management labor) increases from

[15]In comparison, with a standard CES production function with $\varphi = 0$, the median return to variety in my model is 0.2.

Table 10: Real GDP standard deviations and the curvature of management cost function

	(a)	(b)	(c)
Curvature of management cost function	2	3.71	6
Real GDP with EMA	2.44%	1.88%	1.72%
Real GDP w/o EMA	1.51%	1.51%	1.51%
With EMA / without EMA	1.61	1.24	1.14

Note: Real GDP is GDP deflated by the GDP deflator. Standard deviation is that of hp-filtered log real GDP. Column (b) shows simulation results using the baseline parameter values calibrated and estimated as in Sections 5.1, in which the curvature of management cost function is 3.71. Columns (a) and (c) are the results using the baseline parameter values except that the curvature of management cost function are 2 and 6, respectively. "*With EMA*" is the model with extensive margin adjustments; "*Without EMA*" is the model simulated without extensive margin adjustments. "*With EMA / without EMA*" is the ratio between the real GDP standard deviations of the models with and without extensive margin adjustments. Calculation uses the supply chain management cost data from APQC.

2 to 3.71, and then 6, the real GDP standard deviation decreases from 2.44% to 1.72%. Because varying the management cost curvature does not affect the results of models without extensive margin adjustments. All the decrease in aggregate fluctuations due to a rising marginal management cost is due to dampened amplification effects. Extensive margin adjustments amplify aggregate fluctuations by three-fifths when the curvature of the cost function is 2, while the amplification effect reduces to less than one-seventh when the curvature is 6. Combining Tables 9 and 10, we see that both the return to input variety and cost of managing them significantly affect the amplification effect of extensive margin adjustments and thus the aggregate fluctuation.

Table 11: Real GDP standard deviations and input elasticity of substitution

	(a)	(b)	(c)
Input elasticity of substitution	0.175	0.8	2
Real GDP with EMA	1.88%	1.90%	1.95%
Real GDP w/o EMA	1.51%	1.53%	1.56%
With EMA / without EMA	1.24	1.24	1.25

Note: Real GDP is GDP deflated by the GDP deflator. Standard deviation is that of hp-filtered log real GDP. Column (b) shows simulation results using the baseline parameter values calibrated and estimated as in Section 5.1, in which the intermediate input elasticity of substitution is 0.175. Columns (a) and (c) are the results using the baseline parameter values except that the intermediate input elasticity of substitution are 0.8 and 2, respectively. "*With EMA*" is the model with extensive margin adjustments; "*Without EMA*" is the model simulated without extensive margin adjustments. "*With EMA / without EMA*" is the ratio between the real GDP standard deviations of the models with and without extensive margin adjustments. Calculation uses the supply chain management cost data from APQC.

In contrast, Table 11 shows that aggregate fluctuations are affected little by the intermediate input elasticity of substitution and the curvature of supply chain management cost function if the parameters including industry productivity processes are at their baseline values. From columns (a) to (c), an increase of input elasticity from the baseline 0.175 to 0.8 and 2 raises the aggregate fluctuations in both models with and without extensive margin adjustments. However, changes are not as large as those when the return to variety changes. Real GDP standard deviation increases from 1.88% to 1.95% when the elasticity becomes 2 instead of 0.175. More importantly, the amplification effect of extensive margin adjustments moves little from 1.24 to 1.25. This is because although the elasticity of substitution affects the output co-movements among different industries, the changes in industry-level outputs offset each other when aggregated.

5.5 The Role of the Production Network Structure in Amplification

Column (c) of Table 8 shows that the production network structure matters for how large extensive margin adjustments can amplify aggregate fluctuations. In this section, I use two designed networks to look further into the role of the network structure in amplification.

Table 12: Real GDP standard deviations in Different Production Network Structures

	(a) Baseline	(b) Shortened Network	(c) Exchange Mfg. and Retail
$Real\ GDP\ with\ EMA$	1.88%	1.14%	1.54%
$Real\ GDP\ w/o\ EMA$	1.51%	1.03%	1.31%
$With\ \ EMA\ /\ without\ EMA$	1.24	1.10	1.18

Note: Real GDP is GDP deflated by the GDP deflator. Standard deviation is that of hp-filtered log real GDP. Column (a) shows simulation results using the baseline parameter values calibrated and estimated as in Section 5.1. Columns (b) and (c) are the results if the intermediate input share of Manufacturing is set to 10e-6 and if Manufacturing and Retail's input shares are exchanged, respectively. Other parameters are set to their baseline values. "$Without\ EMA$" is the model simulated without extensive margin adjustments. "$With\ EMA\ /\ without\ EMA$" is the ratio between the real GDP standard deviations of the models with and without extensive margin adjustments. Calculation uses the supply chain management cost data from APQC.

Table 12 compares the real GDP standard deviations in the baseline model (column a) with those in two alternative production networks. Column (b) reduces the intermediate input share of Manufacturing to 10e-6, while proportionally increases its capital and labor input shares. This decrease in intermediate input share is essentially shortening the production network. The result says that the amplification of aggregate fluctuations due to extensive margin adjustments decreases from the baseline 0.24 to only 0.1. Notice that Manufacturing accounts for much less than half of the total sales or value-added in the economy. However, cutting its upstream supply chains reduces the amplification effect of EMA by more than half for the following reason. First, in the production network, the lengths of different industries' supply chains depend on Manufacturing's supply chain. When Manufacturing loses its upstream suppliers, those industries which source from Manufacturing also see their supply chains shortened. As I discussed in Section 4.4, a shortened supply chain reduces the reinforcement of extensive margin adjustments on different linkages of the supply chain. As a result, the amplification effect decreases.

Column (c), on the other hand, exchange the intermediate, capital, and labor cost shares of Manufacturing and Retail. The intermediate input cost shares of Manufacturing and Retail are 71% and 43%, and their shares in the aggregate consumption are 16% and 11%, respectively. As a result, exchanging their intermediate input shares essentially reduces the weighted numbers of supply chains in the economy. Following a similar logic above, we should expect the amplification effect of extensive margin adjustments to decrease, which is confirmed by a drop in the amplification effect from 0.24 to 0.18 in column (c).

6 A Unified Labor Market and Alternative Extensive Margin Calibration

In this section, I present the results in a model with a unified labor market or with alternative extensive margin calibration. First, I show that EMA can amplify aggregate fluctuations even if the production and management labor markets are unified, as long as the labor supply is elastic enough. Second, I show that calibrating and estimating the model without EMA or with alternative measures change the amplification effect of EMA.

6.1 A Unified Labor Market and Frisch Elasticity of Labor Supply

A major deviation of my paper from the literature of the production network with costly linkages is segmented markets of production and management labors. In this section, I check whether extensive margin adjustments still amplify aggregate fluctuations when there is a unified market of both production and management labor. In this economy, extensive margin adjustments depend not only on the curvature of the management cost function but also on the Frisch elasticity of labor supply. When the labor supply is inelastic as in Lim (2018) and Taschereau-Dumouchel (2019), we should expect the adjustments in supply chain relationships to be smaller than those with high elasticity of labor supply.

In this model with a unified labor market, the problem of firm i in industry n becomes

$$\pi_{n,i,t} = \max_{\substack{p_{n,i,t},k_{n,i,t},l_{n,i,t},V_{n,i,s,t} \\ \{x_{n,i,s,t}(m)\}_{s,m}}} (p_{n,i,t} - mc_{n,i,t})q_{n,i,t} - w_t \sum_s v^{\eta}_{n,i,s,t}, \tag{49}$$

the representative household's problem becomes

$$\begin{aligned} &\max_{C_t(s^t),K_{t+1}(s^t),L_t(s^t)} \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(logC_t(s^t) - \frac{L_t(s^t)^{1+\epsilon_L}}{1+\epsilon_L}\right) \\ \text{s.t.}\quad & P^C_t(s^t)C_t(s^t) + P^K_t(s^t)\Big(K_{t+1}(s^t) - K_t(s^t)\Big) \leq w_t(s^t)L_t(s^t) \\ & + r_t(s^t)K_t(s^t) + \sum_{n\in S}\int_0^1 \pi_{n,i,t}(s^t)di \qquad \forall t \; and \; s^t, \end{aligned} \tag{50}$$

and the labor market clearing condition becomes

$$L_t(s^t) = \sum_{n\in S}\int_0^1 \Big(l_{n,i,t}(s^t) + \sum_{s\in S} v^{\eta}_{n,i,s,t}(s^t)\Big)di. \tag{51}$$

In particular, I allow the management cost function to have a separate curvature than the Frisch elasticity of substitution to match the response of input supplier numbers to input expenditures in the data.

Table 13: Real GDP standard deviations in a unified labor market

	(a)	(b)	(c)
Frisch elasticity of labor supply	0.001	1	2
Parameters			
Median return to variety	0.159	0.188	0.200
Curvature of management cost function	2.6	3.1	3.3
Results			
Real GDP with EMA	0.840%	1.367%	1.737%
Real GDP w/o EMA	0.836%	1.264%	1.511%
With EMA / without EMA	1.00	1.08	1.15

Note: Real GDP is GDP deflated by the GDP deflator. Standard deviation is that of hp-filtered log real GDP. Production and management labors used by all industries are in a unified market. Models are re-calibrated and re-estimated. From columns (a) to (c), the Frisch elasticities of labor supply are 0.001, 1, and 2, respectively. "*With EMA*" is the model with extensive margin adjustments; "*Without EMA*" is the model simulated without extensive margin adjustments. "*With EMA / without EMA*" is the ratio between the real GDP standard deviations of the models with and without extensive margin adjustments. Calculation uses the supply chain management cost data from APQC.

Table 13 shows the real GDP standard deviations in a unified labor market and with different

Frisch elasticity of labor supply. I experiment with three values of Frisch elasticity: 0.001, 1, and 2. I set Frisch elasticity to 0.001 to approximate the inelastic labor supply as in Lim (2018) and Taschereau-Dumouchel (2019). 1 and 2 are two values between 0.75 and 3, the former of which was suggested by Chetty et al. (2011) and the latter matches the inter-temporal substitution elasticity found by macroeconomists (Prescott, 2006). Whenever I change the elasticity of labor supply, I treat the model as a new one and recalibrate and re-estimate the model. When the labor supply elasticity decreases from 2 to 0.001, real GDP standard deviation decreases a lot from 1.74% to 0.84% (with extensive margin adjustments). This decrease is consistent with the fact that labor input fluctuations explain a large share of aggregate fluctuations. When labor supply becomes less and less elastic, aggregate fluctuations become smaller and smaller. More importantly, the elasticity of labor supply also controls the amount of labor used for managing input varieties. As a result, a less inelastic labor supply leads to less adjustment in the extensive margin and thus a smaller amplification of industry productivity shocks by extensive margin adjustments. When labor supply is almost inelastic, extensive margin hardly adjust and amplify aggregate fluctuations. The real GDP standard deviation is almost the same with and without extensive margin adjustments.[16] On the other hand, when we increase the Frisch elasticity to 2, the conventional number used in macroeconomics, extensive margin adjustments still amplify aggregate fluctuations by more than one-seventh even though the labor market is unified. This indicates that relaxing the assumption of either a unified labor market or inelastic labor supply leads to the amplification of aggregate fluctuations by extensive margin adjustments.

Another takeaway from the above model with a unified labor market is the reason why allowing an additional margin to adjust leads to a seemingly negative consequence, i.e., a larger aggregate fluctuation. In bad times, the marginal productivity of labor is low. Due to the disutility of labor supply, households would rather stay at home than work. As a result, increasing unemployment indeed maximizes households' utility. Similarly, during busts, cutting supply chain relationships leads to a further decrease in productivity and output. However, it is optimal for the household to reduce the supply of management labor because of the low marginal productivity compared to the disutility of labor supply. Thus, allowing the extensive margin adjustments in inputs results in larger aggregate fluctuations, but higher welfare.

6.2 Alternative Ways of Calibration and Estimation

The above sections discuss the importance of extensive margin adjustments on aggregate fluctuations assuming that we take into consideration the extensive margin when calibrating and estimating

[16] Notice that the re-estimated ratio of the return and cost of input varieties does not change much with labor supply elasticity because the supply chain management cost share in total revenue is kept at the baseline value of 3%.

the model. However, the alternative possibility is that we entirely ignore the extensive margin and estimate the model without extensive margin or use the intermediate input measures by the BEA. The following table compares the aggregate fluctuations under these alternative settings.

Table 14: Real GDP standard deviations with alternative estimation and intermediate input measure

	(a) Baseline	(b) Ignore EMA	(c) BEA intermediate input measure
Parameters			
Input elasticity of substitution	0.175	0.176	0.185
Curvature of management cost function	3.71	3.70	3.62
Results			
Real GDP with EMA	1.88%		
Real GDP w/o EMA	1.51%	1.40%	1.04%
Without EMA / with EMA	0.81	0.74	0.55

Note: Real GDP is GDP deflated by the GDP deflator. Standard deviation is that of hp-filtered log real GDP. Column (a) shows simulation results using the baseline parameter values calibrated and estimated as in Section 5.1. Columns (b) and (c) are the results when the model is calibrated and estimated without extensive margin and using the BEA intermediate input measures, respectively. "*Without EMA*" is the model simulated without extensive margin adjustments. "*Without EMA / with EMA*" is the ratio between the real GDP standard deviations of the models without extensive margin adjustments in different settings and the baseline model with extensive margin adjustments. Calculation uses the supply chain management cost data from APQC.

Table 14 shows the real GDP standard deviations in the baseline model (column a) and in models calibrated and estimated without extensive margin and with the BEA intermediate input measures, respectively. We can see that ignoring the extensive margin during the estimation stage leads to a lower real GDP standard deviation than in a model where the parameters are estimated with extensive margin, but extensive margin adjustments are shut down during simulation.[17] If I use the BEA intermediate input measures in the calibration of industry productivity processes, the real GDP standard deviation is even smaller at 0.55 of that in the baseline model. Because the BEA intermediate input measures use Fisher indexes, the contribution of a supplier industry's input to the productivity of the producer is proportional to the share of inputs that the producer sources from the supplier industry (as if using the Cobb–Douglas function). When input elasticity of substitution is less than one, different industries' inputs are more complementary than in a Cobb–Douglas

[17]The industry productivity shocks are less volatile in a model estimated and calibrated without the extensive margin than in the baseline model. However, this is because in the data, industry-level productivity shocks are negatively correlated with intermediate inputs. Assumptions like sticky-price are needed to generate this negative correlation (e.g. Basu et al., 2006). I follow the production network literature not to include these mechanisms. As a result, I take the difference between the real GDP standard deviations with and without extensive margin adjustments under the same baseline parameter values as the amplification effect of extensive margin adjustments.

function. Thus, changes in the inputs from a supplier industry generate larger fluctuations in productivity when the elasticity of substitution is less than one. In the data, this extra movement in the productivities of intermediate inputs makes calibrated industry-level total factor productivities more volatile. As a result, the baseline model generates a larger aggregate fluctuation than that in a model calibrated using the BEA intermediate input measures.

To summarize, I estimate and simulate the full model in this section and study how extensive margin adjustments affect shock transmission and amplify aggregate fluctuations. The size of the return to variety, the marginal cost of managing suppliers, and the production network structure all matter for the size of the amplification. Moreover, the amplification effect of extensive margin adjustments exists even if production and management labor are in a unified labor market as long as the labor supply is elastic enough. Extensive margin adjustments explain a large share of the aggregate fluctuation generated by industry productivity shocks, which is around half of that due to the fluctuations in labor input.

7 Conclusion

The business cycle literature has been studying mechanisms that amplify small variations in productivity to generate large aggregate fluctuations. In this paper, I show how producers' choices of input variety numbers amplify productivity shocks and aggregate fluctuations. I rely on both micro-level empirical evidence and quantitative analysis of a multi-industry real business cycle model to explore the effects of extensive margin adjustments.

First, I document three facts about the number of input suppliers using firm-level supply chain relationship data among US firms. These facts indicate a return to more input varieties and a cost of managing varieties. Second, based on the empirical findings, I develop a real business cycle model with firms' choices of input variety numbers in addition to input quantities. The model shows that extensive margin adjustments amplify productivity shocks and thus the aggregate fluctuation. Finally, I extend the model to a multi-industry environment with a production network for estimation using industry-level data and simulation. I estimate the multi-industry model using indirect inference. Simulation of the model shows that with extensive margin adjustments, industry productivity shocks generate a real GDP standard deviation of 1.88%, which is one-fourth larger than that in a conventional model with fixed numbers of input varieties. I also find that the production network structure not only affects the transmission of shocks but also decides how extensive margin adjustments on different linkages of the supply chain complement and reinforce each other.

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A Data

This appendix describes the details of the construction of the data used for the empirical evidence of Section 2. Some of them are also used in the indirect inference estimation of Sections 5 and 6.

A.1 FactSet Revere Database and Firm-level Supply Chain

The industry-level supplier number is constructed using the FactSet Revere database. This database is currently the most comprehensive database for customer-supplier relationships among US firms. The relationship information[18] are collected systematically from public sources such as SEC 10-K annual filings, investor presentations, and press releases reported by either the customer or supplier firms. Compared to the commonly used Compustat Customer Segment database which only includes major customers that contribute to more than 10% of a firm's revenue[19], FactSet Revere provides a much less truncated set of suppliers. The broader coverage results in more accurate numbers of suppliers with more substantial variation across firms/industries and over time, which are important for the analysis of adjustments on the extensive margin.

The FactSet supply chain dataset gathers 871,547 customer-supplier relationship records between 596,678 pairs of firms from 2003-2019. Each relationship record includes the start and end dates of the relationship. For industry-level analysis, I need firms' industry classifications. I use the first six digits of firms' Committee on Uniform Securities Identification Procedures (CUSIP) numbers to merge with Compustat Capital IQ Historic Segment data to obtain firms' NAICS classifications. I classify firms into fifteen two-digit NAICS industries, which include Agriculture, forestry, fishing, and hunting, Mining, Utilities, Construction, Manufacturing, Wholesale trade,

[18] I use the FactSet Revere dataset subscribed by the Wharton Research Data Services (WRDS).

[19] Public-traded companies are required to report their major customers in accordance with Financial Accounting Standards No. 131.

Retail trade, Transportation and warehousing, Information, Finance and insurance, real estate, rental, and leasing, Professional and business services, Educational services, Health care and social assistance, Arts, entertainment, recreation, accommodation, and food services, Other services, except government, and Government.

In terms of sample coverage, I focus on supply chain relationships among US firms because my paper is about domestic production networks. Also, FactSet collects historical information only back to 2011 for European firms, 2013 for Japanese firms, and 2015 for other Asian firms. I avoid sample truncation by focusing on US firms. Furthermore, the reasons for the exit of a firm from their database include but do not limit to bankruptcy. Thus, I exclude the drop of suppliers due to reasons other than producers' choices. I drop the entire record of a customer-supplier relationship if its termination year coincides with the exit year of either the customer or the supplier side of this relationship. I also merge my supply chain dataset with the SDC Platinum dataset and exclude those relationships which terminate due to a merger or an acquisition of either the customer or the supplier firm. For the same pair of customer and supplier firms, some relationship records overlap or seem consecutive. I consider these records to be consecutive and combine them into one relationship if the gap between them is less than one year. The above selection of the sample leaves me with 61,937 customer-supplier relationships between 58,254 pairs of firms, involving 5246 customer firms and 5148 supplier firms.

A.2 Numbers of Suppliers, Input Expenditures, and Supply Chain Management Costs

With the relationship records in hand, I construct industry-level and firm-level numbers of suppliers. On the industry level, I construct the panel data of supplier numbers between each pair of industries. In each year and for each pair of customer industry A and supplier industry B, I count the number of relationships in which the customer and the supplier firms belong to industries A and B, respectively. The fifteen industries I use lead to 256 inter-industry observations in each year. I denote by $v_{ind,ns,t}$ the number of suppliers that customer industry n sources from in industry s in year t. Denote customer industry n's total supplier number by $v_{ind,n,t} = \sum_{s'} v_{ind,ns',t}$. On the firm level, I count each firm i's total number of suppliers in year t and denote it by $v_{firm,i,t}$.

The industry-level intermediate input expenditures, real output, and capital input data are from the BEA. I calculate industry-by-industry input expenditures based on the BEA Input-Output (before redefinition) Make and Use tables. The annual Use tables are commodity-by-industry, which show industry n's use of commodity s in dollars for each pair of industry n and commodity s. The annual Make tables are industry-by-commodity, which show industry m's production of commodity s in dollars for each pair of industry m and commodity s. Since the mapping between industries and commodities is not one-to-one, I use the Make tables to convert commodity-by-industry intermediate input expenditures to industry-by-industry following Tian (2019). Denote by $pxv_{ind,ns,t}$ industry n's use of industry s good as intermediate inputs in dollars at t and by $pxv_{ind,n,t} = \sum_{s'} pxv_{ind,ns',t}$ industry n's total intermediate input expenditures. I deflate intermediate input expenditures using the GDP deflator. On the other hand, real output, and capital and labor inputs by industry are obtained from the BEA Industry Accounts. Chain-type quantity indexes for gross output by industry are used as real output and denoted by $q_{ind,n,t}$ for industry n in year t. Chain-type quantity indexes for the net stock of private (government) fixed assets by industry are used as capital inputs and denoted by $k_{ind,n,t}$ for industry n in year t. The number of hours is computed using the Industry Hours and Employment data of the BLS Industry Productivity and Costs database. Hours include those of paid employees, the self-employed (partners and proprietors), and unpaid family workers. The number of hours by industry from the BLS is used as labor input and denoted by $l_{ind,n,t}$ for industry n in year t.

On the firm-level, there is no intermediate input expenditure data between customers and suppliers. Thus, I use a firm's cost of goods sold (i.e., "all costs directly allocated by the company to production, such as material, labor, and overhead," and denoted by $cogs_{firm,i,t}$ for firm i at t) and its number of employee $l_{firm,i,t}$ (Compustat code emp) from CRSP/Compustat Merged Fundamentals Annual to construct intermediate input expenditures similar to Keller and Yeaple (2009). Firm i's real intermediate input in period t is $xv_{firm,i,t} = (cogs_{firm,i,t} - l_{firm,i,t} * w_{ind,n,t})/p_{ind,n,t}$. $w_{ind,n,t}$ and $p_{ind,n,t}$ are the BEA wages and salaries per full-time equivalent employee and the chain-type intermediate input price index of firm i's (up to four-digit) NAICS industry n, respectively. Firm i's sales (Compustat code $sale$) at period t is deflated by the BEA chain-type gross output price index of firm i's (up to four-digit) NAICS industry n to construct the real sales $sales_{firm,i,t}$. I construct investment and capital stock using the gross plant, property, and equipment (Compustat code $ppegt$) and the net plant, property, and equipment (Compustat code $ppent$) following Ottonello and Winberry (2018). In particular, I set the first value of $k_{firm,i,t+1}$ to $ppegt_{firm,i,t}$ in the first period in which this variable is reported in CRSP/Compustat since 1950. From this period onwards, I compute investment (net of depreciation) as $inv_{firm,i,t} = ppent_{firm,i,t} - ppent_{firm,i,t-1}$ and capital stock as $k_{firm,i,t+1} = k_{firm,i,t} + inv_{firm,i,t}$. The initial capital stock and investment each period are both deflated by the (up to four-digit) NAICS industry-level (fixed asset) investment price index. This investment price index is constructed as the ratio of the BEA historical-cost price to the chain-type quantity indexes for investment in private fixed assets. If a firm has a missing observation of $inv_{firm,i,t}$ located between two periods with nonmissing observations, I impute the missing observation using a linear interpolation.

In addition to input expenditures, the supply chain management cost (i.e., the cost of managing suppliers) also affects the choice of supplier numbers. Supply chain management is an internal operational process, for which there is no direct data on the dynamics of the cost. Thus, I utilize the OES dataset to construct a proxy for supply chain management costs. The OES includes a semiannual survey on occupational employment and wage rates for workers in US nonfarm establishments. This dataset provides occupational estimates of employment and wages by industry. The estimates are based on six panels of survey data collected over a three-year cycle. As a result, the estimate at any point depends on the survey data in the past three years. For example, data collected in May 2016 are combined with data collected in November 2015, May 2015, November 2014, May 2014, and November 2013 to provide an estimate for May 2016. I use the two-digit NAICS industry-level employment estimate of the occupation “Purchasing Agents, Except Wholesale, Retail, and Farm Products" (in the May of each year) to proxy the supply chain management cost, and denote it by $purch\_emp_{ind,n,t}$ for industry n in year t. The definition of this occupation by the BLS is “purchasing machinery, equipment, tools, parts, supplies, or services necessary for the operation of an establishment, and purchasing raw or semi-finished materials for manufacturing.” This occupation excludes "Buyers and Purchasing Agents, Farm Products" and "Wholesale and Retail Buyers, Except Farm Products," which involve purchases for resale. Thus, the occupation “Purchasing Agents, Except Wholesale, Retail, and Farm Products" is only related to the production process. The OES survey spans 1988 to 2018. However, its estimates by NAICS industry started in 2002. Also, the data for the occupation “Purchasing Agents, Except Wholesale, Retail, and Farm Products" ended in 2016. As a result, I focus on the period from 2003 to 2016.

A.3 Weighted Industry-level Total Number of Suppliers and Input Quantity

Because different supplier industries' shares in customer industry n's input expenditures are different, extensive margin adjustments in different supplier industries by the customer industry vary in the importance. Therefore, I weight a customer industry's number of suppliers and input quantity per supplier in each supplier industry by the intermediate input shares. I define extensive and intensive margin adjustments as $dlnv_{ind,n,t}$ and $dlnx_{ind,n,t}$, respectively. The adjustments on the two margins are computed as follows:

$$dlnv_{ind,n,t} = \sum_s \frac{\omega_{ind,ns,t-1} + \omega_{ind,ns,t}}{2} d\ln v_{ind,ns,t}$$

$$dlnx_{ind,n,t} = \sum_s \frac{\omega_{ind,ns,t-1} + \omega_{ind,ns,t}}{2} d\ln x_{ind,ns,t}$$

where $\omega_{ind,ns,t} = pxv_{ind,ns,t} \Big/ \sum_{s'} pxv_{ind,ns',t}$ is the share of industry s' goods in industry n's total intermediate input expenditures in year t. In particular, I use the average input shares of two adjacent years $t-1$ and t because some input shares change a lot from year to year.

A.4 Estimate the Return to Variety using Olley-Pakes and Levinsohn-Petrin

In Olley-Pakes estimations, I use investment $lninv_{firm,i,t}$ to proxy for the unobserved time-varying productivity shock (not due to extensive margin adjustments). $lninv_{firm,i,t}$ is constructed as the share of current period's investment (net of depreciation) in the capital stock at the beginning of the period, i.e., $lninv_{firm,i,t} = \ln k_{firm,i,t+1} - \ln k_{firm,i,t}$. I deviate from Olley and Pakes (1996) in constructing this proxy because net investment better reflects firms' decisions after observing the productivity shock. Moreover, I use the share of net investment in capital stock rather than the log investment because the net investment can be negative sometimes. I also control for four-digit NAICS industry-level dummies because the price indexes used for different industries' firms are not comparable, leading to different intercepts for different firms. In Levinsohn-Petrin estimations, I follow Levinsohn and Petrin (2003) to use real intermediate inputs $\ln xv_{firm,i,t}$ as a proxy for the unobserved time-varying productivity shock. Because $\ln xv_{firm,i,t}$ is used as a proxy, its coefficient is not identified, and there is no estimate for it. I use the R package *prodest* of Rovigatti (2018) to estimate the production functions.

B Alternative Motivation behind Extensive Margin Adjustments

In this section, I describe possible alternative motivations behind extensive margin adjustments. Then, I provide evidence that these motives are not the driving forces behind EMAs.

An immediate motivation behind pro-cyclical adjustments of supplier numbers is the capacity constraint. During booms, firms expand their input purchases, which may hit the capacity constraints of individual suppliers'. As a result, firms are forced to source from more suppliers. Although capacity constraint is unobservable, this story has an implication which can be tested. We know the capacity constraint is more likely to be hit when input expenditures increase than when they decrease. As a result, firms' supplier numbers increase more in expansions than they decrease in contraction if extensive margin adjustments are mainly driven by the capacity constraint. Thus, although input expenditures (sales) per supplier increase when input expenditures (sales) rise, the increase should be less than the decrease when input expenditures (sales) fall. This implication is tested by estimating the following two equations:

$$d\ln(\frac{pxv_{ind,n,t}}{v_{ind,n,t}}) = \beta_{101} d\ln(pxv_{ind,n,t}) + \beta_{102} dummy_{pxv} d\ln(pxv_{ind,n,t}) + \epsilon_{10,n,t}, \quad (52)$$

$$d\ln(\frac{sales_{firm,i,t}}{v_{firm,i,t}}) = \beta_{111} d\ln(sales_{firm,i,t}) + \beta_{112} dummy_{sales} d\ln(sales_{firm,i,t}) + \epsilon_{11,n,t}, \quad (53)$$

where $dummy_{pxv}$ and $dummy_{sales}$ are equal to one when input expenditures (sales) increase, and are zero otherwise. β_{102} and β_{112} are expected to be positive if the capacity constraint is the major reason behind extensive margin adjustments.

Table 15: Input Expenditure (Sales) per Supplier vs. Input Expenditure (Sales)

VARIABLES	(a) Input Expenditure per Supplier	(b) Sales per Supplier
$pxv_{ind,n,t}$	0.507***	
	(0.190)	
$dummy_{pxv}*pxv_{ind,n,t}$	0.496	
	(0.367)	
$sales_{firm,i,t}$		0.875***
		(0.024)
$dummy_{sales}*sales_{firm,i,t}$		0.0763**
		(0.034)
Observations	195	14,627
R^2	0.181	0.289

Note: Data are annual from 2003 to 2016. The dependent variables are industry's input expenditure per supplier and firm's sales per supplier in columns (a) and (b), respectively. $pxv_{ind,n,t}$ is the industry's total intermediate input expenditure, and $sales_{firm,i,t}$ is the firm's sales. $dummy_{pxv}$ and $dummy_{sales}$ are equal to one when input expenditures and sales increase, respectively, and are zero otherwise. All variables except the dummies are log-differenced. Input expenditures and sales are deflated using the GDP deflator. Industry and firm fixed effects are controlled in columns (a) and (b), respectively. Standard errors are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Columns (a) and (b) of Table 15 show the results of regressions 52 and 53. Unsurprisingly, when the total input expenditures (of a customer industry) and the sales (of a customer firm) increase by 1%, input expenditures and sales per supplier increase by 0.5% and 0.88%, respectively. The coefficients of the two interaction terms, on the other hand, are insignificantly and significantly positive in columns (a) and (b), respectively. The positive coefficients indicate that the rise in suppliers in expansions is larger than the fall in contractions. The asymmetry of extensive margin adjustments is in the opposite direction of what the capacity constraint story implies. As a result, I argue that the capacity constraint is not the driving force of extensive margin adjustments in the data.

Another motivation for using more suppliers is to increase competition among suppliers and reduce costs. If this motivation is the driving force of extensive margin adjustments, I should find pro-cyclical markups due to the pro-cyclical adjustments of supplier numbers in the data. However, Bils, Klenow, and Malin (2018) found that price markups estimated using intermediate input shares are counter-cyclical. Thus, I rule out the possibility that increasing competition to lower cost is the driving force of extensive margin adjustments.

The other reason for extensive margin adjustments can be risk mitigation. Firms are worried about supply chain disruptions, which can result in operation and production interruptions and significant losses. For example, Carvalho, Nirei, Saito, and Tahbaz-Salehi (2016) documented a 1.2% decline in Japan's gross output due to supply chain disruptions in the year following the 2011 Great East Japan Earthquake. However, if the risk concern is the driving motive of adjustments in supplier numbers, the adjustments are more likely to be counter-cyclical due to a higher risk in downturns.

With the above reasonings, I assume a productivity gain of more input suppliers (varieties) in my model.

C Math Appendix

This appendix presents the definition of the competitive equilibrium in Section 4.3, the simplified multi-industry model in Section 4.4, and the proof of the propositions in the main text.

C.1 Competitive Equilibrium Definition of the Full Model

Definition 2. *A competitive equilibrium is composed of firms' output prices* $\left\{p_{n,i,t}(s^t)\right\}_{n,i}$*, consumption and capital good prices* $P_t^C(s^t)$ *and* $P_t^K(s^t)$*, the wages of production and management labors* $w_t(s^t)$ *and* $\{w_{mng,n,t}(s^t)\}_n$*, and the capital rental rate* $r_t(s^t)$*; allocations* $\left\{L_t(s^t), C_t(s^t), K_{t+1}(s^t), C_t^P(s^t), K_t^P(s^t)\right\}$, $\{L_{mng,n,t}(s^t)\}_n$, $\left\{x_{n,i,s,t}(m)(s^t)\right\}_{n,i,s,m}$, $\left\{V_{n,i,s,t}(s^t)\right\}_{n,i,s}$, $\left\{c_{n,i,t}(s^t), k^P_{n,i,t}(s^t), k_{n,i,t}(s^t), l_{n,i,t}(s^t), q_{n,i,t}(s^t)\right\}_{n,i}$ $\forall$ *t and* s^t *such that:*

1. *Given consumption and capital good prices, wage, and capital rental rate,* $\left\{C_t(s^t), K_{t+1}(s^t), L_t(s^t)\right\}_{t,s^t}$ *and* $\{L_{mng,n,t}(s^t)\}_n$ *solve the representative household's problem in equation 35.*

2. *Given other firms' prices, the wages, and the capital rental rate,* $\left\{x_{n,i,s,t}(m)(s^t)\right\}_{s,m}$, $\left\{V_{n,i,s,t}(s^t)\right\}_{s}$, $k_{n,i,t}(s^t)$, $l_{n,i,t}(s^t)$, $p_{n,i,t}(s^t)$ *solve firm's problem in equation 29* $\forall$ *n, i, t, and* s^t*.*

3. *Given all firms' prices,* $\left\{c_{n,i,t}(s^t)\right\}_{n,i}$ *and* $\left\{k^P_{n,i,t}(s^t)\right\}_{n,i}$ *solve consumption and capital good*

producers' problems in equations 32 and 33 $\forall$ t and s^t.

4. The market of each firm's goods clears, i.e., $\forall\ n,\ i,\ t,$ and s^t
$q_{n,i,t}(s^t) = k^P_{n,i,t}(s^t) + c_{n,i,t}(s^t) + \sum_{s\in S}\int_0^1 \mathbb{1}\{i \in V_{s,m,n,t}\}x_{s,m,n,t}(i)(s^t)dm.$

5. Consumption and capital good markets clear, i.e., $\forall$ t and s^t
$C_t(s^t) = C^P_t(s^t),\ K^P_t(s^t) = K_{t+1}(s^t) - K_t(s^t) + \sum_n \delta_n \int_{i\in[0,1]} k_{n,i,t}(s^t)di.$

6. Capital rental and labor market clear, i.e., $\forall$ t and s^t
$K_t(s^t) = \sum_{n\in S}\int_{i\in[0,1]} k_{n,i,t}(s^t)di \quad$ *and* $\quad L_t(s^t) = \sum_{n\in S}\int_{i\in[0,1]} l_{n,i,t}(s^t)di.$

7. Supply chain management labor markets clear, i.e., $\forall\ n,\ t,$ and s^t
$L_{mng,n,t}(s^t) = \int_0^1 \sum_s v_{n,i,s,t}(s^t)di.$

C.2 Simplified Multi-industry Model

The problem of industry n representative firm is

$$\begin{aligned}
\pi_n = &\max_{p_n,l_n,\{x_{ns},v_{ns}\}_s} (p_n - mc_n)q_n - w_{mng,n}\sum_s v_{ns} \\
\text{s.t.}\quad & Z_n X_n^{\alpha} l_n^{1-\alpha} \geq q_n \\
& X_n = \left[\sum_{s\in S}\omega_{ns}\left(v_{ns}^{1+\varphi_s} x_{ns}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma(\epsilon_x-1)}{(\gamma-1)\epsilon_x}}\right]^{\frac{\epsilon_x}{\epsilon_x-1}} \\
& \quad = \left[\sum_{s\in S}\omega_{ns}\left(v_{ns}^{\frac{1+\varphi_s\gamma}{\gamma-1}}\left(v_{ns}x_{ns}\right)\right)^{\frac{\epsilon_x-1}{\epsilon_x}}\right]^{\frac{\epsilon_x}{\epsilon_x-1}},
\end{aligned} \tag{54}$$

where all industries share the same intermediate input share α. Notice that (as if) facing a monopolistic competitive demand, firms' choose prices $p_n = \gamma/(\gamma-1)mc_n$. I still make the Assumption 1 that industry representative firms take as given the prices of other industries and the wages of production and management labors. The marginal cost of industry n firm is

$$mc_n = Z_n^{-1}\left[\sum_{s\in S}\omega_{ns}^{\epsilon_x}\left(v_{ns}^{\frac{1+\varphi_s\gamma}{1-\gamma}} p_s\right)^{1-\epsilon_x}\right]^{\frac{\alpha}{1-\epsilon_x}} w^{1-\alpha}\cdot constant_{mc'} \tag{55}$$

where $constant_{mc'} = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$.

On the household side, there is a representative household who combines goods from all industries into the aggregate consumption good and consumes. Assume that the representative household incurs only disutility from supplying management labor, then its problem is:

$$
\begin{aligned}
\max_{\{c_n, L_{mng,n}\}_n,\ L} \quad & log\Big(C - \sum_{n\in S} L_{mng,n}^{\eta}\Big) \\
s.t. \quad P^C C &= wL + \sum_{n\in S}(\pi_n + w_{mng,n}L_{mng,n}) \\
C &= \left(\sum_{n=1}^{N} \xi_n^c c_n^{\frac{\epsilon_c - 1}{\epsilon_c}}\right)^{\frac{\epsilon_c}{\epsilon_c - 1}}
\end{aligned}
\tag{56}
$$

where the price of the aggregate consumption good price is[20]

$$
P^C = \left(\sum_{n=1}^{N} \left(\xi_n^c\right)^{\epsilon_c} p_n^{1-\epsilon_c}\right)^{\frac{1}{1-\epsilon_c}} \tag{57}
$$

I further assume total production labor supply is fixed and $L = \bar{L} = 1$. Prices are normalized so that the wage of production labor $w = 1$.

C.3 Proof of the Propositions

Proposition 1. *Under Assumptions 1 to 3, there is a unique symmetric equilibrium which satisfies equilibrium definition 1, and all firms behave symmetrically such that,*

1. Firms choose the same input quantity and number of varieties, price, and production and management labor, i.e., $x_i(m) = x\ \forall\, i, m \in V_i$ and $v_i = v,\ p_i = p,\ l_i = l\ \forall\, i$.

2. The consumption good producer uses the same quantity of goods from each firm, i.e., $c_i = c\ \forall\, i$.

Proof. Under Assumption 3, firms are randomly chosen as suppliers. Thus, they have no incentive to lower prices to be selected. Then under the monopolistic competition assumption in Assumption

[20] In computation, I normalize $\{v_{ns}\}_{n,s}$, $\{x_{ns}\}_{n,s}$, and $\{c_n\}_n$ with their steady-state values so that $\{\omega_{ns}\}_{n,s}$ is the long-run share of industry s goods in the intermediate input of industry n, and ξ_n^C is the long-run share of industry n good in the aggregate consumption good.

2, the demand for firm i's goods is

$$\begin{aligned}
q_i(p_i) &= \left(p_i/P^C\right)^{-\gamma} C + \int_0^1 \mathbb{1}\{i \in V_m\} v_m^{\varphi\gamma} \left(p_i/\Phi_m\right)^{-\gamma} X_m dm \\
&= p_i^{-\gamma} \Big[\left(P^C\right)^{\gamma} C + \int_0^1 \mathbb{1}\{i \in V_m\} v_m^{\varphi\gamma} \Phi_m^{\gamma} X_m dm \Big] \\
&= p_i^{-\gamma} \Big[\left(P^C\right)^{\gamma} C + \mathbb{E}\Big(\int_0^1 \mathbb{1}\{i \in V_m\} v_m^{\varphi\gamma} \Phi_m^{\gamma} X_m dm \Big) \Big] \\
&= p_i^{-\gamma} \Big[\left(P^C\right)^{\gamma} C + \int_0^1 \mathbb{E}\{i \in V_m\} v_m^{\varphi\gamma} \Phi_m^{\gamma} X_m dm \Big] \\
&= p_i^{-\gamma} \Big[\left(P^C\right)^{\gamma} C + \int_0^1 Pr\{i \in V_m\} v_m^{\varphi\gamma} \Phi_m^{\gamma} X_m dm \Big] \\
&= p_i^{-\gamma} \Big[\left(P^C\right)^{\gamma} C + \int_0^1 v_m^{1+\varphi\gamma} \Phi_m^{\gamma} X_m dm \Big] = p_i^{-\gamma} \tilde{D},
\end{aligned} \tag{58}$$

where aggregate demand for firm i's goods is defined as $\tilde{D} \equiv \left(P^C\right)^{\gamma} C + \int_0^1 v_m^{1+\varphi\gamma} \Phi_m^{\gamma} X_m dm$, $\Phi_m = \left(\int_{V_m} v_m^{\varphi\gamma} p_i^{1-\gamma} di \right)^{1/(1-\gamma)}$ is the composite intermediate input price of firm m. $\left(p_i/P^C\right)^{-\gamma} C$ is firm i's consumption demand while $\int_0^1 v_m^{1+\varphi\gamma} \Phi_m^{\gamma} X_m dm$ is the intermediate input demand. The second equality holds due to the law of large numbers and the fourth one holds due to random selection for suppliers. Due to these two assumptions, $\left[\left(P^C\right)^{\gamma} C + \int_0^1 v_m^{1+\varphi\gamma} \Phi_m^{\gamma} X_m dm \right]$ is unaffected by p_i and the subscript of $\tilde{D}$ can be omitted.

Now I derive the marginal cost of firm i from the cost minimization problem 15 under Assumption 1. The Lagrange function of the problem is

$$\begin{aligned}
\mathcal{L} &= \int_{m \in V_i} p_m x_i(m) dm + w l_i + w_{mng} v_i + mc_i \left(q_i - Z X_i^{\alpha} l_i^{1-\alpha} \right) \\
&= \int_{m \in V_i} p_m x_i(m) dm + w l_i + w_{mng} v_i + mc_i \left[q_i - Z \left(\int_{m \in V_i} v_i^{\varphi} x_i(m)^{\frac{\gamma-1}{\gamma}} dm \right)^{\frac{\gamma\alpha}{\gamma-1}} l_i^{1-\alpha} \right],
\end{aligned}$$

where mc_i is the Lagrange multiplier and thus the shadow price of output, or the marginal cost.

First-order conditions w.r.t. l_i and $x_i(m)$ yield

$$l_i: \quad w = (1-\alpha) mc_i Z X_i^{\alpha} l_i^{-\alpha}, \tag{59}$$

$$x_i(m): \quad p_m = mc_i Z \alpha X_i^{\alpha-1} l_i^{1-\alpha} \left(\int_{m \in V_i} v_i^{\varphi} x_i(m)^{\frac{\gamma-1}{\gamma}} dm \right)^{\gamma/(\gamma-1)-1} v_i^{\varphi} x_i(m)^{-1/\gamma}. \tag{60}$$

Raise both sides of equation 60 to the power of $1-\gamma$, then multiply by $v_i^{\varphi\gamma}$, and integrate over V_i, we have

$$\int_{V_i} v_i^{\varphi\gamma} p_m^{1-\gamma} dm = \left\{ mc_i Z\alpha X_i^{\alpha-1} l_i^{1-\alpha} \left[\int_{m\in V_i} v_i^{\varphi} x_i(m)^{\frac{\gamma-1}{\gamma}} dm \right]^{\gamma/(\gamma-1)-1} \right\}^{1-\gamma} \int_{V_i} v_i^{\varphi} x_i(m)^{(\gamma-1)/\gamma} dm \tag{61}$$

Raise both sides of equation 61 to the power of $1/(1-\gamma)$, we have

$$\left(\int_{V_i} v_i^{\varphi\gamma} p_m^{1-\gamma} dm \right)^{1/(1-\gamma)} = \alpha mc_i Z X_i^{\alpha-1} l_i^{1-\alpha}. \tag{62}$$

Denote $\Phi_i \equiv \left(\int_{V_i} v_i^{\varphi\gamma} p_m^{1-\gamma} dm \right)^{1/(1-\gamma)}$, which is the price of the composite intermediate input used by firm i. Raise both sides of equations 59 and 62 to the powers of α and $1-\alpha$, respectively, combine them, and rearrange, we have

$$mc_i = Z^{-1} \Phi_i^{\alpha} w^{1-\alpha} \alpha^{-\alpha} (1-\alpha)^{\alpha-1}, \tag{63}$$

which is exactly equation 16.

Using the envelope theorem, we derive the optimal price set by firms.

$$\begin{aligned}
\frac{\partial \pi_i}{\partial p_i} &= \frac{\partial (p_i - mc_i) p_i^{-\gamma}}{\partial p_i} \left[\left(P^C\right)^{\gamma} C + \int_0^1 v_m^{1+\varphi\gamma} \Phi_m^{\gamma} X_m dm \right] = 0 \\
\Leftrightarrow \quad & (1-\gamma) p_i^{-\gamma} = -\gamma mc_i p_i^{-\gamma-1} \\
\Leftrightarrow \quad & p_i = \frac{\gamma}{\gamma-1} mc_i,
\end{aligned}$$

which is equation 19.

Now we derive the optimal choice of input varieties. Denote $\frac{\partial y}{\partial m}$ as the partial effect of adding input variety m on any variable y, and use the envelope theorem, we have

$$\begin{aligned}
\frac{\partial \pi_i}{\partial m} &= -\frac{\partial mc_i}{\partial m} q_i - w_{mng} \\
\Leftrightarrow -w_{mng} &= \frac{\partial mc_i}{\partial m} q_i, \\
&= \alpha mc_i q_i \left(\int_{V_i} v_i^{\varphi\gamma} p_m^{1-\gamma} dm \right)^{-1} \frac{1}{1-\gamma} \left(v_i^{\varphi\gamma} p_m^{1-\gamma} + \varphi\gamma \int_{V_i} v_i^{\varphi\gamma-1} p_m^{1-\gamma} dm \right).
\end{aligned} \tag{64}$$

Integrate both sides of equation 64 on the set V_i and rearrange, we have

$$\begin{aligned} w_{mng} v_i &= -\alpha m c_i q_i \Big(\int_{V_i} v_i^{\varphi\gamma} p_m^{1-\gamma} dm \Big)^{-1} \frac{1}{1-\gamma} \Big(\int_{V_i} v_i^{\varphi\gamma} p_m^{1-\gamma} dm + \varphi\gamma \int_{V_i} v_i^{\varphi\gamma} p_m^{1-\gamma} dm \Big), \\ &= \frac{1+\varphi\gamma}{\gamma-1} \alpha m c_i q_i = \frac{1+\varphi\gamma}{\gamma-1} \int_{V_i} p_m x_i(m) dm. \end{aligned} \tag{65}$$

Now we derive the optimal choices by the household. Write down the Lagrange function of household's problem 17

$$\begin{aligned} \mathcal{L} &= log\Big(C - L_{mng}^{\eta}\Big) + \lambda\Big(wL + \int_0^1 \pi_i di + w_{mng} L_{mng} - P^C C\Big) \\ &= log\left[\Big(\int_0^1 c_i^{\frac{\gamma-1}{\gamma}} di\Big)^{\frac{\gamma}{\gamma-1}} - L_{mng}^{\eta}\right] + \lambda\left[wL + \int_0^1 \pi_i di + w_{mng} L_{mng} - P^C \Big(\int_0^1 c_i^{\frac{\gamma-1}{\gamma}} di\Big)^{\frac{\gamma}{\gamma-1}}\right], \\ &= log\left[\Big(\int_0^1 c_i^{\frac{\gamma-1}{\gamma}} di\Big)^{\frac{\gamma}{\gamma-1}} - L_{mng}^{\eta}\right] + \lambda\left[wL + \int_0^1 \pi_i di + w_{mng} L_{mng} - \int_0^1 p_i c_i di\right]. \end{aligned} \tag{66}$$

First-order conditions of equation yield

$$c_i : \quad \Big(C - L_{mng}^{\eta}\Big)^{-1} \Big(\int_0^1 c_i^{\frac{\gamma-1}{\gamma}} di\Big)^{\frac{1}{\gamma-1}} c_i^{-\frac{1}{\gamma}} = \lambda p_i = \lambda P^C \Big(\int_0^1 c_i^{\frac{\gamma-1}{\gamma}} di\Big)^{\frac{1}{\gamma-1}} c_i^{-\frac{1}{\gamma}}, \tag{67}$$

$$L_{mng} : \quad \Big(C - L_{mng}^{\eta}\Big)^{-1} \eta L_{mng}^{\eta-1} = \lambda w_{mng}. \tag{68}$$

From the second equality of equation 67, we have the price of the aggregate consumption

$$P^C = \left(\int_0^1 p_i^{1-\gamma}\right)^{\frac{1}{1-\gamma}}. \tag{69}$$

Combine equations 67 and 68, we have

$$w_{mng} = P^C \eta L_{mng}^{\eta-1}. \tag{70}$$

Now I prove the existence of a symmetric equilibrium by constructing such an equilibrium that satisfies equilibrium conditions 58, 59, 60, 62, 63, 65, 67, 68, 70, 19, and market clearing conditions, and all variables (except V_i which differ in the elements but share the same measure $v_i = v$) take the same value for each firm.

First, I show that in a symmetric equilibrium, any individual firm has no incentive to deviate from the symmetric solutions. Suppose in an equilibrium, all firms set the same price $p_i = p \ \forall i \in [0, 1]$.

According to the first-order condition w.r.t. input quantity, 60, $x_i(m) = x_i\ \forall m \in V_i\ \&\ \forall i \in [0,1]$. Also,

$$\Phi_i = \left(\int_{V_i} v_i^{\varphi\gamma} p^{1-\gamma} dm\right)^{1/(1-\gamma)} = v_i^{\frac{1+\varphi\gamma}{1-\gamma}} p, \tag{71}$$

$$mc_i = Z^{-1} w^{1-\alpha} \alpha^{-\alpha} (1-\alpha)^{\alpha-1} v_i^{\alpha\frac{1+\varphi\gamma}{1-\gamma}} p^{\alpha}. \tag{72}$$

Replace the mc_i and q_i in equation 65 with equations 58 and 72, we have

$$\begin{aligned} w_{mng} v_i &= v_i^{\alpha\frac{1+\varphi\gamma}{1-\gamma}} \frac{1+\varphi\gamma}{\gamma-1} Z^{-1} w^{1-\alpha} \big(\alpha/(1-\alpha)\big)^{1-\alpha} p^{\alpha-\gamma} \left[\left(P^C\right)^{\gamma} C + \int_0^1 v_m^{1+\varphi\gamma} \Phi_m^{\gamma} X_m dm\right] \\ \Leftrightarrow v_i^{1-\alpha\frac{1+\varphi\gamma}{1-\gamma}} &= w_{mng}^{-1} \frac{1+\varphi\gamma}{\gamma-1} Z^{-1} w^{1-\alpha} \big(\alpha/(1-\alpha)\big)^{1-\alpha} p^{\alpha-\gamma} \left[\left(P^C\right)^{\gamma} C + \int_0^1 v_m^{1+\varphi\gamma} \Phi_m^{\gamma} X_m dm\right], \end{aligned} \tag{73}$$

where the left hand side of the second equation is the same for all firms. As a result, $v_i = v\ \forall i \in [0,1]$. In turn, according to equations 71 and 72, $\Phi_i = \Phi$ and $mc_i = mc$ for all $i \in [0,1]$. From equation 19, we confirm that $p_i = \gamma/(\gamma-1)mc = p\ \ \forall i \in [0,1]$. Thus, given that all suppliers set the same price, the marginal costs of all firms will be the same, so do their prices, which is intrinsically coherent. In turn, $x_i(m) = x,\ \ \forall\, i \in [0,1]\ \&\ m \in V_m$ according to equation 65. In other words, given that other firms in the economy choose the same price, the optimal solution to a firm's problem is to set the same price, use the same number of input varieties, and use the same input quantity from each chosen variety as other firms. Thus, each individual firm will not deviate from the symmetric equilibrium solution.

Because $q = ZX_i^{\alpha} l_i^{1-\alpha} = Z\big(X_i/l_i\big)^{\alpha} l_i$, and combined with equation 59, we have

$$\begin{aligned} q &= \frac{w l_i}{(1-\alpha)mc} \\ \Leftrightarrow l_i &= \frac{(1-\alpha)mc \cdot q}{w} \equiv l, \quad \forall\ i \in [0,1], \end{aligned} \tag{74}$$

and in turn

$$X_i = \left(\frac{w}{(1-\alpha)mc \cdot Z}\right)^{1/\alpha} l \equiv X, \quad \forall\ i \in [0,1]. \tag{75}$$

Because $X = v^{\frac{\gamma(1+\varphi)}{\gamma-1}} x_i$, $x_i(m) = x_i = x\ \forall m \in V_i\ \&\ \forall i \in [0,1]$. Then equation 58 becomes

$$q_i(p) = p^{-\gamma}\left[\left(P^C\right)^{\gamma} C + \Phi^{\gamma} X v^{1+\varphi\gamma}\right] = q, \quad \forall\, i \in [0,1], \tag{76}$$

The symmetric equilibrium can be constructed as and must be a set of 15 different groups of variables

$$x_i(m) = x \quad \forall\, i \in [0,1] \;\&\; m \in V_i,$$
$$v_i = v, \quad q_i = q, \quad p_i = p, \quad mc_i = mc, \quad X_i = X, \quad l_i = l, \quad c_i = c, \quad \Phi_i = \Phi \quad \forall\, i \in [0,1],$$
$$\text{and } w,\; w_{mng},\; L_{mng},\; C,\; P^C,\; \lambda$$

which satisfy the following 15 equations: first order conditions,

$$\Phi = \alpha mc \cdot q/X, \tag{77}$$
$$w_{mng} = \frac{1+\varphi\gamma}{\gamma-1} px, \tag{78}$$
$$p = \frac{\gamma}{\gamma-1} mc, \tag{79}$$
$$w = (1-\alpha) mc \cdot q/l, \tag{80}$$
$$w_{mng} = P^C \eta L_{mng}^{\eta-1}, \tag{81}$$
$$\left(C - L_{mng}^{\eta}\right)^{-1} = \lambda P^C, \tag{82}$$

market clearing conditions,

$$l = \bar{L} = 1, \tag{83}$$
$$q = c + vx, \tag{84}$$
$$v = L_{mng}, \tag{85}$$

and aggregators and the production function,

$$X = v^{\frac{(1+\varphi)\gamma}{\gamma-1}} x, \tag{86}$$
$$\Phi = v^{\frac{1+\varphi\gamma}{1-\gamma}} p, \tag{87}$$
$$C = c, \tag{88}$$
$$P^C = p, \tag{89}$$
$$w = 1, \tag{90}$$
$$q = ZX^{\alpha} l^{1-alpha}. \tag{91}$$

According to the proof of Proposition 3, the solution of q is unique, $v = \left((1+\varphi\gamma)\alpha q/(\eta\gamma)\right)^{1/\eta}$ is unique, and $x = \alpha(\gamma-1)q/\gamma$ is also unique. As a result, the other variables in the equilibrium also have unique solutions. Thus, we can conclude that a symmetric equilibrium exists and is unique. □

Proposition 2. *Under Assumptions 1 to 3, $\forall n, s \in S$ and in the (symmetric) equilibrium, the number of input varieties used satisfies*

$$\frac{1+\varphi\gamma}{\gamma-1}pxv = P^C\eta v^\eta. \tag{20}$$

Proof. In a symmetric equilibrium, combine equations 65, 81, and 85, we have

$$\frac{1+\varphi\gamma}{\gamma-1}pxv = w_{mng}v = P^C\eta v^\eta.$$

□

Proposition 3. *Under Assumptions 1 to 3 and in the (symmetric) equilibrium, the aggregate output and consumption in the economy, $Q = q$ and C can be represented as functions of only the productivity Z,*

$$Q = Z^{\frac{1}{1-\alpha-\alpha\frac{1+\varphi\gamma}{\eta(\gamma-1)}}} \cdot constant_q, \tag{24}$$

$$C = \Big(1-\alpha\frac{\gamma-1}{\gamma}\Big)Z^{\frac{1}{1-\alpha-\alpha\frac{1+\varphi\gamma}{\eta(\gamma-1)}}} \cdot constant_q, \tag{25}$$

where $constant_q = \left(\left(\alpha\frac{1+\varphi\gamma}{\eta(\gamma-1)}\right)^{\alpha\frac{1+\varphi\gamma}{\eta(\gamma-1)}}\left(\frac{\gamma-1}{\gamma}\alpha\right)^{\alpha}\right)^{\frac{1}{1-\alpha-\alpha\frac{1+\varphi\gamma}{\eta(\gamma-1)}}}$.

Proof. Using Proposition 2 and $P^C = p$,

$$p\eta v^\eta = \frac{1+\varphi\gamma}{\gamma-1}pxv = \frac{1+\varphi\gamma}{\gamma-1}\alpha pq\frac{\gamma-1}{\gamma} = \frac{1+\varphi\gamma}{\gamma}\alpha pq,$$

$$\Leftrightarrow v = \Big(\frac{1+\varphi\gamma}{\eta\gamma}\alpha q\Big)^{1/\eta}. \tag{92}$$

On the other hand,

$$pxv = \alpha\frac{\gamma-1}{\gamma}pq,$$

$$\Leftrightarrow vx = \alpha\frac{\gamma-1}{\gamma}q. \tag{93}$$

Replace the vx and v in the production function 91 with equations 92 and 93, we have

$$q = ZX^\alpha l^{1-\alpha} = Z(vx)^\alpha v^{\frac{1+\varphi\gamma}{\gamma-1}\alpha},$$

$$= Z\Big(\alpha\frac{\gamma-1}{\gamma}q\Big)^\alpha\Big(\frac{1+\varphi\gamma}{\eta\gamma}\alpha q\Big)^{\frac{1+\varphi\gamma}{\eta(\gamma-1)}\alpha}.$$

Rearrange, we have

$$q^{1-\alpha-\frac{1+\varphi\gamma}{\eta(\gamma-1)}\alpha} = ZX^{\alpha}l^{1-\alpha} = Z(vx)^{\alpha}v^{\frac{1+\varphi\gamma}{\gamma-1}\alpha},$$
$$\Leftrightarrow Q = q = Z^{\frac{1}{1-\alpha-\alpha\frac{1+\varphi\gamma}{\eta(\gamma-1)}}} \cdot constant_q,$$

where $constant_q = \left(\left(\alpha\frac{1+\varphi\gamma}{\eta(\gamma-1)}\right)^{\alpha\frac{1+\varphi\gamma}{\eta(\gamma-1)}}\left(\frac{\gamma-1}{\gamma}\alpha\right)^{\alpha}\right)^{\frac{1}{1-\alpha-\alpha\frac{1+\varphi\gamma}{\eta(\gamma-1)}}}$.

According to the market clearing condition for goods, 84,

$$C = c = q - vx = q - \alpha\frac{\gamma-1}{\gamma}q = \left(1-\alpha\frac{\gamma-1}{\gamma}\right)q,$$
$$= \left(1-\alpha\frac{\gamma-1}{\gamma}\right)Z^{\frac{1}{1-\alpha-\alpha\frac{1+\varphi\gamma}{\eta(\gamma-1)}}} \cdot constant_q.$$

□

Proposition 4. *Under Assumptions 1, 4, and 5, there is a symmetric equilibrium which satisfies equilibrium definition 1, and all firms within an industry behave symmetrically such that* $\forall\, t$ *and* s^t,

1. Firms within an industry choose the same input quantity and number of varieties, price, production and management labor, and capital input, i.e., $x_{n,i,s,t}(m)(s^t) = x_{ns,t}(s^t)\,\forall n,\ i,\ s\,\&\, m \in V_{n,i,s,t}$; *and* $v_{n,i,s,t}(s^t) = v_{ns,t}(s^t)\,\forall n\ \&\ i$; $p_{n,i,t}(s^t) = p_{n,t}(s^t),\ l_{n,i,t}(s^t) = l_{n,t}(s^t),\ k_{n,i,t}(s^t) = k_{n,t}(s^t)\,\forall n$ *and* i.

2. The consumption good producer uses the same quantity of goods from each firm in an industry n*, so does the capital good producer, i.e.,* $c_{n,i,t}(s^t) = c_{n,t}(s^t),\ k^P_{n,i,t}(s^t) = k^P_{n,t}(s^t)\,\forall n$ *and* i.

Proof. Under Assumption 5, firms are randomly chosen as suppliers from an industry. Thus, they have no incentive to lower prices to be selected. Then under the monopolistic competition assumption in Assumption 4, the demand for industry n, firm i's goods at time t is

$$\begin{aligned}
q_{n,i,t}(p_{n,i,t}) =& p_{n,i,t}^{-\gamma_n}\Bigg\{\Bigg[P_t^C (C_t^P)^{\frac{1}{\epsilon_c}}\xi_n^C\Big(\int_0^1 c_{n,i',t}^{\frac{\gamma_n-1}{\gamma_n}}di'\Big)^{\frac{\gamma_n}{\gamma_n-1}\frac{\epsilon_c-1}{\epsilon_c}-1}\Bigg]^{\gamma_n} + \Bigg[\sum_s\int_0^1 \mathbb{1}\{i\in V_{s,m,n,t}\} \\
&\Bigg(\alpha_{x,s}mc_{s,m,t}q_{s,m,t}X_{s,m,t}^{\frac{1-\epsilon_x}{\epsilon_x}}\omega_{sn}\Big(\int_{V_{s,m,n,t}} v_{s,m,n,t}^{\varphi_n}x_{s,m,n,t}(i')^{\frac{\gamma_n-1}{\gamma_n}}di'\Big)^{\frac{\gamma_n}{\gamma_n-1}\frac{\epsilon_x-1}{\epsilon_x}-1} v_{s,m,n,t}^{\varphi_n}\Bigg)^{\gamma_n}dm\Bigg]\Bigg\} \\
=& p_{n,i,t}^{-\gamma_n}\Bigg\{\Bigg[P_t^C (C_t^P)^{\frac{1}{\epsilon_c}}\xi_n^C\Big(\int_0^1 c_{n,i',t}^{\frac{\gamma_n-1}{\gamma_n}}di'\Big)^{\frac{\gamma_n}{\gamma_n-1}\frac{\epsilon_c-1}{\epsilon_c}-1}\Bigg]^{\gamma_n} + \\
&\Bigg[\sum_s\int_0^1 v_{s,m,n,t}^{1+\varphi_n\gamma_n}\Bigg(\alpha_{x,s}mc_{s,m,t}q_{s,m,t}X_{s,m,t}^{\frac{1-\epsilon_x}{\epsilon_x}}\omega_{sn}\Big(\int_{V_{s,m,n,t}} v_{s,m,n,t}^{\varphi_n}x_{s,m,n,t}(i')^{\frac{\gamma_n-1}{\gamma_n}}di'\Big)^{\frac{\gamma_n}{\gamma_n-1}\frac{\epsilon_x-1}{\epsilon_x}-1}\Bigg)^{\gamma_n}dm\Bigg]\Bigg\} \\
\equiv& p_{n,i,t}^{-\gamma_n}\tilde{D}_{n,t},
\end{aligned} \tag{94}$$

where aggregate demand for industry n, firm i's goods is defined as $\tilde{D}_{n,t}$. The first term in the braces is firm i's consumption demand while the second term is the intermediate input demand. Due to random selection for suppliers and the law of large numbers, $\tilde{D}_{n,t}$ is unaffected by $p_{n,i,t}$ and also the subscript i can be omitted following the derivation of equation 58.

Now I derive the marginal cost of industry n, firm i from the cost minimization problem 30 under Assumption 1. The Lagrange function of the problem is

$$\mathcal{L} = \sum_{s \in S} \int_{m \in V_{n,i,s,t}} p_{s,m,t} x_{n,i,s,t}(m) dm + (r_t + \delta_n) P_t^K k_{n,i,t} + w_t l_{n,i,t} + w_{mng,n,t} \sum_s v_{n,i,s,t} + \\ mc_{n,i,t} \left(q_{n,i,t} - Z_{n,t} X_{n,i,t}^{\alpha_{x,n}} k_{n,i,t}^{\alpha_{k,n}} l_{n,i,t}^{1-\alpha_{x,n}-\alpha_{k,n}} \right),$$

where $mc_{n,i,t}$ is the Lagrange multiplier and thus the shadow price of output, or the marginal cost.

First-order conditions w.r.t. $k_{n,i,t}$, $l_{n,i,t}$, and $x_{n,i,s,t}(m)$ yield

$$k_{n,i,t}: \quad (r_t + \delta_n) P_t^K = \alpha_{k,n} mc_{n,i,t} q_{n,i,t} / k_{n,i,t}, \tag{95}$$

$$l_{n,i,t}: \quad w_t = (1 - \alpha_{x,n} - \alpha_{k,n}) mc_{n,i,t} q_{n,i,t} / l_{n,i,t}, \tag{96}$$

$$x_{n,i,s,t}(m): \quad p_{s,m,t} = \alpha_{x,n} mc_{n,i,t} q_{n,i,t} X_{n,i,t}^{\frac{1-\epsilon_x}{\epsilon_x}} \omega_{ns} \left(\int_{V_{n,i,s,t}} v_{n,i,s,t}^{\varphi_s} x_{n,i,s,t}(m)^{\frac{\gamma_s - 1}{\gamma_s}} dm \right)^{\frac{\gamma_s}{\gamma_s - 1} \frac{\epsilon_x - 1}{\epsilon_x} - 1} \\ \cdot v_{n,i,s,t}^{\varphi_s} x_{n,i,s,t}(m)^{-\frac{1}{\gamma_s}}. \tag{97}$$

Raise both sides of equation 97 to the power of $1 - \gamma_s$, then multiply by $v_{n,i,s,t}^{\varphi_s \gamma_s}$, integrate over $V_{n,i,s,t}$, and raise both sides to the power of $1/(1 - \gamma_s)$, we have

$$\left(\int_{V_{n,i,s,t}} p_{s,m,t}^{1-\gamma_s} v_{n,i,s,t}^{\varphi_s \gamma_s} dm \right)^{\frac{1}{1-\gamma_s}} = \alpha_{x,n} mc_{n,i,t} q_{n,i,t} X_{n,i,t}^{\frac{1-\epsilon_x}{\epsilon_x}} \omega_{ns} \left(\int_{V_{n,i,s,t}} v_{n,i,s,t}^{\varphi_s} x_{n,i,s,t}(m)^{\frac{\gamma_s - 1}{\gamma_s}} dm \right)^{-\frac{\gamma_s}{\gamma_s - 1} \frac{1}{\epsilon_x}}. \tag{98}$$

Raise both sides of equation 98 to the power of $1 - \epsilon_x$, multiply by $\omega_{ns}^{\epsilon_x}$, sum up by s, and raise both sides to the power of $1/(1 - \epsilon_x)$, we have

$$\left\{ \sum_s \omega_{ns}^{\epsilon_x} \left(\int_{V_{n,i,s,t}} p_{s,m,t}^{1-\gamma_s} v_{n,i,s,t}^{\varphi_s \gamma_s} dm \right)^{\frac{1-\epsilon_x}{1-\gamma_s}} \right\}^{\frac{1}{1-\epsilon_x}} = \alpha_{x,n} mc_{n,i,t} q_{n,i,t} X_{n,i,t}^{\frac{1-\epsilon_x}{\epsilon_x}} X_{n,i,t}^{-\frac{1}{\epsilon_x}} = \alpha_{x,n} mc_{n,i,t} q_{n,i,t} X_{n,i,t}^{-1} \equiv \Phi_{n,i,t}. \tag{99}$$

$\Phi_{n,i,t}$ is the price of the composite intermediate input used by firm i in industry n. Raise both sides of equations 95, 96, and 99 to the powers of $\alpha_{k,n}$, $1 - \alpha_{x,n} - \alpha_{k,n}$, and $\alpha_{x,n}$, respectively, combine them, and rearrange, we have

$$mc_{n,i,t} = Z_{n,t}^{-1} \Phi_{n,i,t}^{\alpha_{x,n}} w_t^{1-\alpha_{x,n}-\alpha_{k,n}} \left((r_t + \delta_n) P_t^K \right)^{\alpha_{k,n}} \alpha_{x,n}^{-\alpha_{x,n}} (1 - \alpha_{x,n} - \alpha_{k,n})^{\alpha_{x,n}+\alpha_{k,n}-1} \alpha_{k,n}^{-\alpha_{k,n}}, \tag{100}$$

which is exactly equation 31.

Using the envelope theorem, we derive the optimal price set by firms.

$$
\begin{aligned}
\frac{\partial \pi_{n,i,t}}{\partial p_{n,i,t}} &= \frac{\partial (p_{n,i,t} - mc_{n,i,t}) p_{n,i,t}^{-\gamma_n}}{\partial p_{n,i,t}} \tilde{D}_{n,t} = 0 \\
\Leftrightarrow \quad & (1-\gamma_n) p_i^{-\gamma_n} = -\gamma_n mc_{n,i,t} p_{n,i,t}^{-\gamma_n - 1} \\
\Leftrightarrow \quad & p_{n,i,t} = \frac{\gamma_n}{\gamma_n - 1} mc_{n,i,t},
\end{aligned}
$$

which is equation 38.

Now we derive the optimal choice of input varieties. $\frac{\partial y}{\partial m}$ is still the partial effect of adding input variety m on any variable y, and use the envelope theorem, we have

$$
\begin{aligned}
\frac{\partial \pi_{n,i,t}}{\partial m} &= -\frac{\partial mc_{n,i,t}}{\partial m} q_{n,i,t} - w_{mng,n,t} \\
\Leftrightarrow -w_{mng,n,t} &= \frac{\partial mc_{n,i,t}}{\partial m} q_{n,i,t}, \\
&= \alpha_{x,n} mc_{n,i,t} q_{n,i,t} \Phi_{n,i,t}^{\epsilon_x - 1} \omega_{ns}^{\epsilon_x} \frac{1}{1-\gamma_s} \left(\int_{V_{n,i,s,t}} p_{s,m,t}^{1-\gamma_s} v_{n,i,s,t}^{\varphi_s \gamma_s} dm \right)^{\frac{1-\epsilon_x}{1-\gamma_s} - 1} \\
&\cdot \left(p_{s,m,t}^{1-\gamma_s} v_{n,i,s,t}^{\varphi_s \gamma_s} + \varphi_s \gamma_s \int_{V_{n,i,s,t}} p_{s,m,t}^{1-\gamma_s} v_{n,i,s,t}^{\varphi_s \gamma_s - 1} dm \right).
\end{aligned}
\tag{101}
$$

Integrate both sides of equation 101 on the set $V_{n,i,s,t}$ and rearrange, we have

$$
w_{mng,n,t} v_{n,i,s,t} = -\alpha_{x,n} mc_{n,i,t} q_{n,i,t} \Phi_{n,i,t}^{\epsilon_x - 1} \omega_{ns}^{\epsilon_x} \frac{1 + \varphi_s \gamma_s}{1-\gamma_s} \left(\int_{V_{n,i,s,t}} p_{s,m,t}^{1-\gamma_s} v_{n,i,s,t}^{\varphi_s \gamma_s} dm \right)^{\frac{1-\epsilon_x}{1-\gamma_s}}. \tag{102}
$$

Multiply both sides of equation 97 by $x_{n,i,s,t}(m)$ and integrate over $V_{n,i,s,t}$, and combine with equation 102, we have

$$
\begin{aligned}
w_{mng,n,t} v_{n,i,s,t} &= -\alpha_{x,n} mc_{n,i,t} q_{n,i,t} \Phi_{n,i,t}^{\epsilon_x - 1} \omega_{ns}^{\epsilon_x} \frac{1 + \varphi_s \gamma_s}{1-\gamma_s} \\
&\cdot \left(\alpha_{x,n} mc_{n,i,t} q_{n,i,t} X_{n,i,t}^{\frac{1-\epsilon_x}{\epsilon_x}} \omega_{ns} \right)^{1-\epsilon_x} \left(\int_{V_{n,i,s,t}} v_{n,i,s,t}^{\varphi_s} x_{n,i,s,t}(m)^{\frac{\gamma_s - 1}{\gamma_s}} dm \right)^{\frac{\gamma_s}{\gamma_s - 1} \frac{\epsilon_x - 1}{\epsilon_x}} \\
&= \frac{1 + \varphi_s \gamma_s}{\gamma_s - 1} \alpha_{x,n} mc_{n,i,t} q_{n,i,t} X_{n,i,t}^{\frac{1-\epsilon_x}{\epsilon_x}} \omega_{ns} \left(\int_{V_{n,i,s,t}} v_{n,i,s,t}^{\varphi_s} x_{n,i,s,t}(m)^{\frac{\gamma_s - 1}{\gamma_s}} dm \right)^{\frac{\gamma_s}{\gamma_s - 1} \frac{\epsilon_x - 1}{\epsilon_x}} \\
&= \frac{1 + \varphi_s \gamma_s}{\gamma_s - 1} \int_{V_{n,i,s,t}} p_{s,m,t} x_{n,i,s,t}(m) dm.
\end{aligned}
\tag{103}
$$

Now we derive the optimal choices by the household. Write down the Lagrange function of household's problem 35

$$\mathcal{L} = \mathbb{E}\Bigg\{\sum_{t=0}^{\infty}\beta^t\Bigg(log\big(C_t(s^t)\big) - \psi\frac{L_t(s^t)^{1+\epsilon_L}}{1+\epsilon_L} - \sum_{n\in S}L^{\eta}_{mng,n,t}(s^t)\Bigg) +$$
$$\sum_{t=0}^{\infty}\lambda_t(s^t)\Bigg[w_t(s^t)L_t(s^t) + r_t(s^t)P^K_t(s^t)K_t(s^t) + \sum_{n\in S}w_{mng,n,t}(s^t)L_{mng,n,t}(s^t) + \sum_{n\in S}\int_0^1\pi_{n,i,t}(s^t)di$$
$$- P^C_t(s^t)C_t(s^t) - P^K_t(s^t)\Big(K_{t+1}(s^t) - K_t(s^t)\Big)\Bigg]\Bigg\}. \tag{104}$$

Normalize the Lagrange multiplier of the budget constraints $\lambda_t(s^t) = \beta^t$, first-order conditions of equation yield $\forall\ t$ & s^t

$$C_t(s^t): \quad C_t(s^t)^{-1} = P^C_t(s^t), \tag{105}$$
$$L_t(s^t): \quad \psi L_t(s^t)^{\epsilon_L} = w_t(s^t), \tag{106}$$
$$K_{t+1}(s^t): \quad P^K_t(s^t) = \beta\mathbb{E}_t\Big[\big(r_{t+1}(s^{t+1}) + 1\big)P^K_{t+1}(s^{t+1})\Big], \tag{107}$$
$$L_{mng,n,t}(s^t): \quad \eta L^{\eta-1}_{mng,n,t} = w_{mng,n,t}. \tag{108}$$

Solve the consumption and capital good producers' cost minimization problems, 32 and 33 similar to solving the aggregate consumption price of 69, we obtain the prices of the consumption and capital goods

$$P^C_t = \left[\sum_{n\in S}(\xi^C_n)^{\epsilon_c}\Big(\int_0^1 p^{1-\gamma_n}_{n,i,t}di\Big)^{\frac{1-\epsilon_C}{1-\gamma_n}}\right]^{\frac{1}{1-\epsilon_C}},$$

and

$$P^K_t = \left[\sum_{n\in S}(\xi^K_n)^{\epsilon_k}\Big(\int_0^1 p^{1-\gamma_n}_{n,i,t}di\Big)^{\frac{1-\epsilon_k}{1-\gamma_n}}\right]^{\frac{1}{1-\epsilon_k}},$$

as in equations 36 and 37.

Now I prove the existence of a symmetric equilibrium by constructing such an equilibrium that satisfies equilibrium conditions 94, 95, 96, 97, 99, 100, 105, 106, 107, 108, 36, 37, 38, and market clearing conditions, and all variables (except $V_{n,i,s,t}$ which differ in the elements but share the same measure $v_{n,i,s,t} = v_{n,s,t}$) take the same values for firms within the same industry.

First, I show that in a symmetric equilibrium, any individual firm has no incentive to deviate from the symmetric solutions. Suppose in an equilibrium, all firms within an industry set the same price $p_{n,i,t} = p_{n,t}\ \forall\ n$ & $i \in [0,1]$. According to the first-order condition w.r.t. input quantity, 97, $x_{n,i,s,t}(m) = x_{n,i,s,t}\ \forall m \in V_{n,i,s,t}$ & $\forall\ n, s$ & $i \in [0,1]$. According to equation 94,

$q_{n,i,t} = p_{n,t}^{-\gamma_n}\tilde{D}_{n,t} = q_{n,t}$ & $\forall\, n$ & $i \in [0,1]$. According to the last equality of equation 103,

$$x_{n,i,s,t}(m) = \frac{\gamma_s - 1}{1+\varphi_s\gamma_s} w_{mng,n,t}/p_{s,t} = x_{n,s,t} \quad \forall m \in V_{n,i,s,t} \;\&\; \forall\, n, s \;\&\; i \in [0,1], \tag{109}$$

which are identical for each pair of industries n and s. Also,

$$\Phi_{n,i,t} = \left\{ \sum_s \omega_{ns}^{\epsilon_x} p_{s,t}^{1-\epsilon_x} v_{n,i,s,t}^{\frac{(1+\varphi_s\gamma_s)(1-\epsilon_x)}{1-\gamma_s}} \right\}^{\frac{1}{1-\epsilon_x}}, \tag{110}$$

$$mc_{n,i,t} = Z_{n,t}^{-1}\Phi_{n,i,t}^{\alpha_{x,n}} w_t^{1-\alpha_{x,n}-\alpha_{k,n}} \left((r_t+\delta_n)P_t^K\right)^{\alpha_{k,n}} \alpha_{x,n}^{-\alpha_{x,n}} (1-\alpha_{x,n}-\alpha_{k,n})^{\alpha_{x,n}+\alpha_{k,n}-1} \alpha_{k,n}^{-\alpha_{k,n}}. \tag{111}$$

Replace the mc_i and q_i in equation 103 with equations 94, 99, and 100 and rearrange, we have

$$\begin{aligned} \sum_{s'} \frac{\gamma_{s'}-1}{1+\varphi_{s'}\gamma_{s'}} v_{n,i,s',t} &= w_{mng,n,t}^{-1}\alpha_{x,n} mc_{n,i,t} q_{n,t} X_{n,i,t}^{\frac{1-\epsilon_x}{\epsilon_x}} X_{n,i,t}^{\frac{\epsilon_x-1}{\epsilon_x}} = w_{mng,n,t}^{-1}\alpha_{x,n} mc_{n,i,t} q_{n,t} \\ &= w_{mng,n,t}^{-1}\alpha_{x,n} Z_{n,t}^{-1}\Phi_{n,i,t}^{\alpha_{x,n}} w_t^{1-\alpha_{x,n}-\alpha_{k,n}} \left((r_t+\delta_n)P_t^K\right)^{\alpha_{k,n}} cst_{mc,n} p_{n,t}^{-\gamma_n}\tilde{D}_{n,t} \equiv T_{n,t}\Phi_{n,i,t}^{\alpha_{x,n}} \\ &= T_{n,t}\left\{ \sum_{s'} \omega_{ns'}^{\epsilon_x} p_{s',t}^{1-\epsilon_x} v_{n,i,s',t}^{\frac{(1+\varphi_{s'}\gamma_{s'})(1-\epsilon_x)}{1-\gamma_{s'}}} \right\}^{\frac{\alpha_{x,n}}{1-\epsilon_x}}. \end{aligned} \tag{112}$$

Using equation 103, we can write $v_{n,i,s',t}\ \forall\, s'$ as functions of $v_{n,i,s,t}$:

$$\begin{aligned} \frac{v_{n,i,s',t}}{v_{n,i,s,t}} &= \frac{(1+\varphi_{s'}\gamma_{s'})(\gamma_s-1)\omega_{ns'}}{(1+\varphi_s\gamma_s)(\gamma_{s'}-1)\omega_{ns}} \left(\frac{x_{n,s',t}}{x_{n,s,t}}\right)^{\frac{\epsilon_x-1}{\epsilon_x}} \frac{v_{n,i,s',t}^{\frac{(1+\varphi_{s'})\gamma_{s'}(\epsilon_x-1)}{(\gamma_{s'}-1)\epsilon_x}}}{v_{n,i,s,t}^{\frac{(1+\varphi_s)\gamma_s(\epsilon_x-1)}{(\gamma_s-1)\epsilon_x}}} \\ \Leftrightarrow v_{n,i,s',t} &= \left[\frac{(1+\varphi_{s'}\gamma_{s'})(\gamma_s-1)\omega_{ns'}}{(1+\varphi_s\gamma_s)(\gamma_{s'}-1)\omega_{ns}} \left(\frac{x_{n,s',t}}{x_{n,s,t}}\right)^{\frac{\epsilon_x-1}{\epsilon_x}} \right]^{1/\tilde{T}_{s'}} v_{n,i,s,t}^{\tilde{T}_s/\tilde{T}_{s'}} \equiv \tilde{O}_{n,s',s,t} v_{n,i,s,t}^{\tilde{T}_s/\tilde{T}_{s'}}, \end{aligned} \tag{113}$$

where $\tilde{T}_s = \left[(\gamma_s-1)\epsilon_x + (1+\varphi_s)\gamma_s(1-\epsilon_x)\right]/(\gamma_s-1) > 0\ \ \forall\, s$ and $\tilde{O}_{n,s',s,t} > 0$. Put equation 113 into equation 112, we have

$$\begin{aligned} \sum_{s'} \frac{\gamma_{s'}-1}{1+\varphi_{s'}\gamma_{s'}} v_{n,i,s',t} &= T_{n,t}\left\{ \sum_{s'} \omega_{ns'}^{\epsilon_x} p_{s',t}^{1-\epsilon_x} v_{n,i,s',t}^{\frac{(1+\varphi_{s'}\gamma_{s'})(1-\epsilon_x)}{1-\gamma_{s'}}} \right\}^{\frac{\alpha_{x,n}}{1-\epsilon_x}} \\ \Leftrightarrow \sum_{s'} \frac{\gamma_{s'}-1}{1+\varphi_{s'}\gamma_{s'}} \tilde{O}_{n,s',s,t} v_{n,i,s,t}^{\tilde{T}_s/\tilde{T}_{s'}} &= T_{n,t}\left\{ \sum_{s'} \omega_{ns'}^{\epsilon_x} p_{s',t}^{1-\epsilon_x} \left(\tilde{O}_{n,s',s,t} v_{n,i,s,t}^{\tilde{T}_s/\tilde{T}_{s'}}\right)^{\frac{(1+\varphi_{s'}\gamma_{s'})(1-\epsilon_x)}{1-\gamma_{s'}}} \right\}^{\frac{\alpha_{x,n}}{1-\epsilon_x}}. \end{aligned} \tag{114}$$

Define a function of $v_{n,i,s,t}$

$$f_{vfun}(v_{n,i,s,t}) = \sum_{s'} \frac{\gamma_{s'}-1}{1+\varphi_{s'}\gamma_{s'}} \tilde{O}_{n,s',s,t} v_{n,i,s,t}^{\tilde{T}_s/\tilde{T}_{s'}} - T_{n,t} \left\{ \sum_{s'} \omega_{ns'}^{\epsilon_x} p_{s',t}^{1-\epsilon_x} \left(\tilde{O}_{n,s',s,t} v_{n,i,s,t}^{\tilde{T}_s/\tilde{T}_{s'}} \right)^{\frac{(1+\varphi_{s'}\gamma_{s'})(1-\epsilon_x)}{1-\gamma_{s'}}} \right\}^{\frac{\alpha_{x,n}}{1-\epsilon_x}}. \tag{115}$$

The first term is strictly increasing in $v_{n,i,s,t}$ while the second term is strictly decreasing in $v_{n,i,s,t}$. As a result, $f'_{vfun}(v_{n,i,s,t}) > 0$. At the same time, $f_{vfun}(0) = -\infty$ and $f_{vfun}(\infty) = \infty$, and $f'_{vfun}(v_{n,i,s,t})$ is continuous in $v_{n,i,s,t}$.[21] Thus, $f_{vfun}(v_{n,i,s,t})$ has a positive solution and it is unique. Because the unique solution only depends on the industry pair n and s, $v_{n,i,s,t} = v_{n,s,t}$ $\forall\, i \in [0,1]$. In turn, according to equations 110 and 111, $\Phi_{n,i,t} = \Phi_{n,t}$ and $mc_{n,i,t} = mc_{n,t}$ for all $i \in [0,1]$. From equation 38, we confirm that $p_{n,i,t} = \gamma_n/(\gamma_n-1)mc_{n,t} = p_{n,t}$ $\forall i \in [0,1]$. Thus, given that all suppliers within an industry set the same price, the marginal costs of all firms within an industry will be the same, so do their prices, which is intrinsically coherent. In other words, given that other firms in the same industry choose the same price, the optimal solution to the problem of a firm in this industry is to set the same price, use the same number of input varieties, and use the same input quantity from each chosen variety as other firms in this industry. Thus, each individual firm will not deviate from the symmetric equilibrium solution.

Under equation 109, $X_{n,i,t} = X_{n,t}$ $\forall\, i \in [0,1]$. Combine equations 95, 96 with 99, we have

$$k_{n,i,t} = \frac{\alpha_{k,n}\Phi_{n,t}X_{n,t}}{\alpha_{x,n}(r_t+\delta_n)P_t^K} = k_{n,t}, \tag{116}$$

$$l_{n,i,t} = \frac{(1-\alpha_{x,n}-\alpha_{k,n})\Phi_{n,t}X_{n,t}}{\alpha_{x,n}w_t} = l_{n,t}. \tag{117}$$

The symmetric equilibrium can be constructed as and must be a set of 21 different groups of variables

$$\begin{aligned}
& x_{n,i,s,t}(m) = x_{n,s,t} \quad \forall\, i \in [0,1]\ \&\ m \in V_i\ \&\ n,s, \\
& v_{n,i,s,t} = v_{n,s,t}, \quad q_{n,i,t} = q_{n,t}, \quad p_{n,i,t} = p_{n,t}, \quad mc_{n,i,t} = mc_{n,t}, \quad X_{n,i,t} = X_{n,t}, \quad l_{n,i,t} = l_{n,t}, \\
& k_{n,i,t} = k_{n,t}, \quad c_{n,i,t} = c_{n,t}, \quad k_{n,i,t}^P = k_{n,t}^P, \quad \Phi_{n,i,t} = \Phi_{n,t} \quad \forall\, i \in [0,1]\ \&\ n, \\
& \text{and } w_t,\ r_t,\ w_{mng,n,t},\ L_t,\ L_{mng,n,t},\ C_t,\ K_t,\ K_t^P,\ P_t^C,\ P_t^K.
\end{aligned}$$

which satisfy the following 21 equations: first order conditions,

$$p_{s,t}x_{n,s,t}v_{n,s,t} = \frac{\gamma_n-1}{\gamma_n}\alpha_{x,n}p_{n,t}q_{n,t}X_{n,t}^{\frac{1-\epsilon_x}{\epsilon_x}}\omega_{ns}v_{n,s,t}^{\frac{(1+\varphi_s)\gamma_s(\epsilon_x-1)}{(\gamma_s-1)\epsilon_x}}x_{n,s,t}^{\frac{\epsilon_x-1}{\epsilon_x}}, \tag{118}$$

[21] In computation, $\{v_{n,i,s,t}\}_{n,i,s,t}$ are normalized to be in $(0,1)$.

$$w_t = (1 - \alpha_{x,n} - \alpha_{k,n}) mc_{n,t} q_{n,t} / l_{n,t}, \tag{119}$$

$$(r_t + \delta_n) P_t^K = \alpha_{k,n} mc_{n,t} q_{n,t} / k_{n,t}, \tag{120}$$

$$w_{mng,n,t} = \frac{1 + \varphi_s \gamma_s}{\gamma_s - 1} p_{n,s,t} x_{n,s,t}, \tag{121}$$

$$p_{n,t} = \frac{\gamma_n}{\gamma_n - 1} mc_{n,t}, \tag{122}$$

$$\psi L_t^{\epsilon_L} = w_t, \tag{123}$$

$$w_{mng,n,t} = \eta L_{mng,n,t}^{\eta - 1}, \tag{124}$$

$$C_t^{-1} = P_t^C, \tag{125}$$

$$P_t^K = \beta \mathbb{E}\left[(r_{t+1} + 1) P_{t+1}^K \right], \tag{126}$$

$$c_{n,t} = \left(\xi_n^C\right)^{\epsilon_c} \left(p_{n,t} / P_t^C\right)^{-\epsilon_c} C_t^P, \tag{127}$$

$$k_{n,t}^P = \left(\xi_n^K\right)^{\epsilon_k} \left(p_{n,t} / P_t^K\right)^{-\epsilon_k} K_t^P, \tag{128}$$

market clearing conditions,

$$\sum_n l_{n,t} = L_t, \tag{129}$$

$$\sum_n k_{n,t} = K_t, \tag{130}$$

$$K_t^P = K_{t+1} - K_t + \sum_n \delta_n k_{n,t}, \tag{131}$$

$$q_{n,t} = c_{n,t} + k_{n,t}^P + \sum_s v_{s,n,t} x_{s,n,t}, \tag{132}$$

$$\sum_s v_{n,s,t} = L_{mng,n,t}, \tag{133}$$

and aggregators and the production function,

$$\Phi_{n,t} = \left(\sum_s \omega_{ns}^{\epsilon_x} p_{s,t}^{1-\epsilon_x} v_{n,s,t}^{\frac{(1+\varphi_s \gamma_s)(1-\epsilon_x)}{1-\gamma_s}} \right)^{\frac{1}{1-\epsilon_x}}, \tag{134}$$

$$q_{n,t} = Z_{n,t} X_{n,t}^{\alpha_{x,n}} k_{n,t}^{\alpha_{k,n}} l_{n,t}^{1-\alpha_{x,n}-\alpha_{k,n}}, \tag{135}$$

$$C_t^P = \left(\sum_{s \in S} \xi_s^C c_{s,t}^{\frac{\epsilon_c - 1}{\epsilon_c}} \right)^{\frac{\epsilon_c}{\epsilon_c - 1}}, \tag{136}$$

$$K_t^P = \left(\sum_{s \in S} \xi_s^K k_{s,t}^{\frac{\epsilon_k - 1}{\epsilon_k}} \right)^{\frac{\epsilon_k}{\epsilon_k - 1}}, \tag{137}$$

$$X_{n,t} = \left(\sum_{s \in S} \omega_{ns} v_{n,s,t}^{\frac{(1+\varphi_s)\gamma_s(\epsilon_x - 1)}{(\gamma_s - 1)\epsilon_x}} x_{n,s,t}^{\frac{\epsilon_x - 1}{\epsilon_x}} \right)^{\frac{\epsilon_x}{\epsilon_x - 1}}, \tag{138}$$

□

Proposition 5. *Under Assumptions 1, 4, and 5 and in the (symmetric) equilibrium, and normalize the Lagrangian multiplier of the budget constraint in household's problem 35 to be β^t in each period t. $\forall n, s \in S$, the number of input varieties used satisfies*

$$\frac{1 + \varphi_s \gamma_s}{\gamma_s - 1} p_{s,t} x_{ns,t} v_{ns,t} = \eta \big(\sum_s v_{ns,t} \big)^{\eta - 1} v_{ns,t}. \tag{39}$$

Proof. Combine equations 103, 108, and market clearing condition 133, and enforce symmetry, we prove the theorem. □

D Indirect Inference Estimation

This appendix describes the indirect inference estimation method used in Sections 5 and 6, as well as the algorithm to implement it. Also, I explain how I calibrate industry productivities within the estimation.

The indirect inference method I use is the partial indirect inference (PII), as advocated by Dridi et al. (2007). Because it is hard to assume such a complicated multi-industry model to be true, I keep my estimation parsimonious with respect to the evidence of extensive margin adjustments, which I would like match. To do so, I estimate only the parameters which can be identified by regression coefficients in my motivational facts while calibrating the others. A parsimonious instrumental model has a lower risk of capturing what goes wrong in the simulated paths due to the misspecification about the parameters not of the most interest.

Based on the identification strategy, the auxiliary model I use in the indirect inference estimation is composed of the following two regressions

$$d \ln\big(\sum_s v_{ns,t}\big) = \beta_1 d \ln\big(\sum_s p_{s,t} x_{ns,t} v_{ns,t}\big) + \epsilon_{1,ns,t}, \tag{139}$$

$$d \ln \left(\frac{p_{s,t} x_{ns,t} v_{ns,t}}{\sum_{s'} p_{s',t} x_{ns',t} v_{ns',t}} \right) = \beta_2 d \ln p_{s,t} + \epsilon_{2,ns,t}. \tag{140}$$

The binding function is

$$\beta = (\beta_1, \beta_2) = b(\eta, \epsilon_x). \tag{141}$$

The indirect inference estimators $\{\tilde{\eta}, \tilde{\epsilon}_x\}$ are defined as the solution to the two-equation system

$$\hat{\beta}_1 = \tilde{\beta}_1(\eta, \epsilon_x), \tag{142}$$

$$\hat{\beta}_2 = \tilde{\beta}_2(\eta, \epsilon_x), \tag{143}$$

where $\{\hat{\beta}_1, \hat{\beta}_2\}$ and $\{\tilde{\beta}_1, \tilde{\beta}_2\}$ are the regression coefficients of equations 139 and 140 using observed data and model-simulated data, respectively. The observed data used for regressions are described in Appendix A.1.

The motivation of using indirect inference estimation is the mis-specification of the regressions used for identification. As mentioned in Section 5.1, the identifications of the parameters governing the return to variety and the management cost curvature depend on each other. Also, the identification of the intermediate input elasticity of substitution is affected by the other two parameters to be identified. It follows that the observed data cannot be used to directly identify the parameters. As a result, I use indirect inference with mis-specified regressions for identification. To be more specific, I replace $\sum_s \left(\left(\frac{1+\varphi_s \gamma_s}{\gamma_s - 1} / \sum_{s'} \frac{1+\varphi_{s'} \gamma_{s'}}{\gamma_{s'} - 1} \right) p_{s,t} x_{ns,t} v_{ns,t} \right)$ in equation 46 with observed $\sum_s pxv_{ns,t}$ and use regression 139 to identify η. Notice that in observed data, input expenditures are $pxv_{ns,t}$ instead of $p_{s,t} x_{ns,t} v_{ns,t}$. I also leave out the unobserved $\Phi_{n,t}$ in equation 48 and use regression 140 to identify ϵ_x. I also exclude the term $d\ln(v_{n,s,t})$ to make the regressions parsimonious in the indirect inference. And ignoring this term does not change much the regression coefficient for identifying ϵ_x. As a result, $\epsilon_{1,ns,t}$ and $\epsilon_{2,ns,t}$ in equations 139 and 140 include those left-out variables and the biases due to mis-specification. The advantage of indirect inference estimation is that I can estimate the key parameters even though the two regressions in the auxiliary model are mis-specified (Smith, 2008). In other word, the densities in the two regressions need not accurately describe the conditional distributions of $d\ln(\sum_s v_{ns,t})$ and $d\ln\left(p_{s,t} x_{ns,t} v_{ns,t} / \left(\sum_{s'} p_{s',t} x_{ns',t} v_{ns',t}\right)\right)$ determined by the true optimality conditions 46 and 48.

To implement indirect inference estimation, I need to simulate the model, which in turn requires industry productivity processes to be calibrated. The calibration must take into consideration extensive margin adjustments because they enter the productivities and thus the conventional total factor productivity. It follows that calibrated industry productivities depend on the key parameters to be estimated. Also, the BEA intermediate input quantity index is a chain-type Fisher index. It does not measure the contribution of intermediate inputs to production because it ignores both the non-unity elasticity of substitution and the extensive margin. As a result, I calibrate industry productivity processes within the estimation. Given a guessed pair of $(\{\varphi_s\}_s, \eta, \epsilon_x)$, I compute log-differenced industry n's productivity as follows

$$d\ln Z_{n,t} = d\ln q_{t,n} - \zeta_n\Big(\alpha_{x,n} d\ln X_{n,t} + \alpha_{k,n} d\ln k_{n,t} + (1-\alpha_{x,n}-\alpha_{k,n}) d\ln l_{n,t}\Big) \tag{144}$$

where ζ_n is the return to scale. Some industries use factors which have almost fixed supply. (e.g., Agriculture uses land.) As a result, these industries have a strong decreasing return to scale. For example, the estimated return to scale of Agriculture is between 0.3 and 0.4. When ζ_n is significantly smaller than one, even if total inputs fluctuate significantly, the output will be relatively stable. As a result, allowing a non-constant return to scale prevents the variations of the calibrated total factor productivities from being entirely driven by variations in the inputs. In implementation, I follow Basu et al. (2006) and Huo et al. (2019) to regress $d\ln q_{n,t}$ on $\alpha_{x,n} d\ln X_{n,t} + \alpha_{k,n} d\ln k_{n,t} + (1 - \alpha_{x,n} - \alpha_{k,n}) d\ln l_{n,t}$, and take the residuals as the log-differenced industry productivities. Then I construct the industry productivity processes using these log-differenced series,

Within the industry productivity calibration above, the composite intermediate input $X_{n,t}$ is

$$\begin{aligned} X_{n,t} &= \left[\sum_{s\in S} \omega_{ns} v_{ns,t}^{\frac{(1-\varphi_s\gamma_s)(\epsilon_x-1)}{(\gamma_s-1)\epsilon_x}} \left(v_{ns,t} x_{ns,t}\right)^{\frac{(\epsilon_x-1)}{\epsilon_x}}\right]^{\frac{\epsilon_x}{\epsilon_x-1}} \\ &= \left[\sum_{s\in S} \omega_{ns} \left(\frac{pxv_{ns,t}\frac{1-\varphi_s\gamma_s}{\gamma_s-1}}{\left(\sum_{s'} \frac{1-\varphi_{s'}\gamma_{s'}}{\gamma_{s'}} pxv_{ns',t}\right)^{\frac{\eta-1}{\eta}}}\right)^{\frac{(1-\varphi_s\gamma_s)(\epsilon_x-1)}{(\gamma_s-1)\epsilon_x}} \left(\frac{pxv_{ns,t}}{p_{s,t}}\right)^{\frac{(\epsilon_x-1)}{\epsilon_x}}\right]^{\frac{\epsilon_x}{\epsilon_x-1}}, \end{aligned} \tag{145}$$

and it is affected by the values of parameters η and ϵ_x to be estimated. As a result, the calibration of industry productivity processes must be incorporated into the indirect inference estimation.

Some intermediate input expenditures among industries are quite volatile. At the same time, their shares in the total intermediate inputs of the customer industry and the total supply of intermediate inputs by the supplier industry are both small. To prevent these volatile input expenditures from dominating the calibration of composite intermediate inputs and industry productivities, I use second-order Taylor expansion to approximate $d\ln X_n$.

I use data from the 1997-2017 BEA Input-Output Tables and Industry Accounts to calibrate the industry productivity processes. Industry-level capital stock and labor are the quantity index of net private (government) fixed asset stock and the number of hours, respectively. Industry-level chain-type gross output price indexes are used for $p_{s,t}$. These data are introduced in Section 2.1.

Besides the three key parameters and the industry productivity processes, the other parameters of the model are calibrated as follows. I set the value of the discount factor β to 0.9709 such that the steady-state real interest rate (i.e., net capital rental rate) is 3%. For the elasticities of substitutions between industries in the production of consumption and capital goods, ϵ_c and ϵ_k, I set both to 0.8.[22] I set the Frisch elasticity of labor supply to 2.0. The intermediate input weight matrix $\Omega = \{\omega_{ns}\}_{n,s}$ is calibrated to its corresponding values in the 2012 BEA Input-Output Accounts because 2012 is the base year of the comprehensive update of industry chain-type indexes. Similar to that in Section 2.1, I combine the BEA Input-Output Make and Use tables to construct the input weight matrix following Tian (2019). The calibration of the other parameters is discussed in Appendix E.

With the calibration and estimation of all parameters introduced above, the complete algorithm of indirect inference estimation and industry productivity calibration is as follows.

Productivity-Calibration-Incorporated Indirect Inference Algorithm

Step 1: Calibrate parameters other than the three key parameters.

Step 2: Search for a pair of η, ϵ_X.

Step 3: Calibrate φ_s to match the steady-state management cost share with that in the data.

Step 4: Given η, ϵ_X and the BEA data, compute industry productivities according to equations 144 and 145 and use them to calibrate $AR(1)$ industry productivity processes (persistency ρ and covariance matrix of industry productivity shocks).

Step 5: Simulate the model for $H * 15$ periods using the calibrated productivity processes.

Step 6: Use regressions 139 and 140 to estimate $\tilde{\beta}_1$, $\tilde{\beta}_2$ with simulated data.

Step 7: Repeat steps (2)-(6) until $\hat{\beta}_1 = \tilde{\beta}_1(\tilde{\eta}, \tilde{\epsilon}_X)$ and $\hat{\beta}_2 = \tilde{\beta}_2(\tilde{\eta}, \tilde{\epsilon}_X)$.

Step 8: Save estimated $\tilde{\eta}$, $\tilde{\epsilon}_X$ and the corresponding industry productivity processes.

E Calibration

This appendix describes the calibration of industry-specific capital depreciation rates, labor, capital, intermediate input shares, and markups. To calibrate these parameters, I first compute some moments using the BEA Input-Output Tables and Industry Accounts spanning 1997 to 2017.

[22] Ngai and Pissarides (2007) argue that "the observed positive correlation between employment growth and relative price inflation across two-digit sectors" supports a low (smaller than 1) substitutability between consumption goods produced by different industries. Also, the preference elasticity of substitution across two-digit manufacturing industries is estimated to be between 0.8 and 1.1 by Oberfield and Raval (2014).

$\forall s \in S$, define:

$$Compensation\ Share_s = \frac{1}{21}\sum_{t=1997}^{2017}\frac{Compensation\ of\ employees_{s,t}}{Gross\ Output_{s,t}}, \tag{146}$$

$$Total\ Intermediate\ Share_s = \frac{1}{21}\sum_{t=1997}^{2017}\frac{Total\ Intermediate_{s,t}}{Gross\ Output_{s,t}}, \tag{147}$$

$$Variable\ Cost\ Share_s = Compensation\ Share_s + Total\ Intermediate\ Share_s, \tag{148}$$

$$Fixed\ Asset\ Share_s = \frac{1}{21}\sum_{t=1997}^{2017}\frac{Fixed\ Asset\ Stock_{s,t}}{Gross\ Output_{s,t}}, \tag{149}$$

$$Depreciation\ of\ Fixed\ Asset\ Share_s = \frac{1}{21}\sum_{t=1997}^{2017}\frac{Depreciation\ of\ Fixed\ Asset\ Stock_{s,t}}{Gross\ Output_{s,t}}. \tag{150}$$

Industry-specific capital depreciation rates are calibrated as

$$\delta_s = \frac{1}{21}\sum_{t=1997}^{2017}\frac{Depreciation\ of\ Fixed\ Asset_{s,t}}{Fixed\ Asset_{s,t}} \tag{151}$$

I define profit shares as

$$\begin{aligned} Profit\ Share_s = 1 - r_{ss} Fixed\ Asset\ Share_s - Variable\ Cost\ Share_s \\ - Depreciation\ of\ Fixed\ Asset\ Share_s \end{aligned} \tag{152}$$

I calibrate γ_s as the inverse of $Profit\ Share_s$, and the markup of industry s is $1/(1 - Profit\ Share_s)$. As can be seen from the formula of industry markups, allowing industry-specific capital depreciation rates is critical for the accurate calibration of industry markups. I calibrate long-run labor and intermediate input cost shares $\alpha_{L,s}$ and $\alpha_{X,s}$ as

$$\alpha_{L,s} = \frac{\gamma_s}{\gamma_s - 1} Compensation\ Share_s \tag{153}$$

$$\alpha_{X,s} = \frac{\gamma_s}{\gamma_s - 1} Total\ Intermediate\ Share_s \tag{154}$$

Then long-run capital cost share is $\alpha_{K,s} = 1 - \alpha_{L,s} - \alpha_{X,s}$.

Long-run consumption and capital good input weights $\{\xi_s^C\}_s$ and $\{\xi_s^K\}_s$ are calibrated as

$$\xi_s^C = \frac{1}{21}\sum_{t=1997}^{2017} \frac{Personal\ consumption\ expenditures_{s,t}}{\sum_{s'} Personal\ consumption\ expenditures_{s',t}}, \tag{155}$$

$$\xi_s^K = \frac{1}{21}\sum_{t=1997}^{2017} \frac{Private\ fixed\ investment_{s,t}}{\sum_{s'} Private\ fixed\ investment_{s',t}}. \tag{156}$$

F Under-estimated Amplification Effects of EMAs in the BEA Data

In this appendix, I describe how the BEA calculates real GDP and industry-level real output, and explain how their calculation leads to an under-estimated return to variety compared to that in the model. Due to this under-estimate, the BEA data could only partially reflect the extensive margin's amplification effects on the volatility of real GDP.

The BEA calculates real GDP and industry outputs by deflating their nominal counterparts with various price indexes. The price indexes used include the Bureau of Labor Statistics (BLS) Producer Price Index (PPI) and Consumer Price Index (CPI), the personal consumption expenditures (PCE) price indexes from the National Income and Product Accounts (NIPAs), which are based on the BLS PPI and CPI, and so on.[23] While the GDP is deflated mainly using the CPI and the PPI, industry outputs are deflated mainly using the PPI and the NIPA PCE prices.

The BEA real GDP and outputs reflect the quality changes of some goods and services (potentially due to extensive margin adjustments) whose corresponding deflators are adjusted for the quality. Some changes in the prices of goods and services surveyed by the BEA or BLS are associated with the changes in the characteristics or quality (e.g., adding remote start or new gear revision to improve fuel economy in vehicles). In some of these cases, quality adjustments are made to the price indexes, allowing them to measure the changes in price levels while holding quality constant. If certain components of the GDP are deflated by quality-adjusted price indexes, the real GDP will embed the quality changes in these components. In the example of Section 1, when Ford introduces a new navigation supplier and makes a car with higher quality and price, its sales value increases. On the other hand, the number of cars sold may not change much. In such a case, using the unadjusted higher price index as the deflator leads to little change in the real GDP. However, the

[23]For detailed price indexes used to deflate GDP and industry output, see "Updated Summary of NIPA Methodologies" and "The 2019 Annual Update of the Industry Economic Accounts" at https://apps.bea.gov/scb/2019/11-november/pdf/1119-nipa-methodologies.pdf and https://apps.bea.gov/scb/2019/11-november/pdf/1119-industry-update.pdf, respectively.

return to variety does lead to productivity gains and a better product for consumers. To account for these improvements, the price needs to be adjusted downwards to measure the price of a car with unchanged and lower quality. As a result, the real GDP calculated using the quality-adjusted car price will be higher than that with an unadjusted price, which in turn correctly measures the quality improvement and the productivity gain due to more input varieties (extensive margin adjustments).

Various methods are used for quality adjustments to the prices. For the CPI, the direct quality adjustment, which includes hedonic regressions and cost adjustments, and the imputation are used by the BLS. The hedonic quality adjustment is used frequently for the CPI (and for some of the PPI). This method decomposes an item into constituent characteristics, obtains estimates of the values of each characteristic, and uses these estimates to adjust the prices.[24] For the PPI, the explicit quality adjustment, which adjusts for the production cost differences (marked up to the selling price) of a change to an item, is the preferred method of quality adjustment.[25]

Unfortunately, only a limited proportion of GDP components are adjusted for quality changes. Groshen, Moyer, Aizcorbe, Bradley, and Friedman (2017) find from 2013 to 2014, only 2% of the items in the CPI were quality-adjusted for their prices. (There are no similar figures for the PPI.) Groshen, Moyer, Aizcorbe, Bradley, and Friedman (2017) also reported that about 33% of the total expenditures in the CPI are eligible for quality adjustments with the hedonic method in 2017. However, most of them are housing-related expenditures. Wasshausen, Moulton, et al. (2006) echoed Groshen, Moyer, Aizcorbe, Bradley, and Friedman (2017) in their 2016 updated appendix that around 17.4% to 20.5% of GDP components were deflated with hedonic-type price indexes. However, around 70% of these components deflated by hedonic prices were structures and rent.

The BLS survey method for CPI further reveals that a large proportion of real GDP does not account for quality changes. When calculating the CPI, the BLS surveys 211 item strata (in each of the 87 primary sampling units/areas), each of which includes one or more entry-level items (ELIs).[26] These ELIs are sampling units which are sampled within each sample outlet. Then in each outlet that the BLS visit, they randomly select a specific item from among all the items the outlet sells that falls within the ELI definition. A match with higher price and quality will be treated as the same item as other items with lower prices and quality in the same item strata. Apparently, the 211 item strata are small relative to the number of differentiated products. As a result, when a match is found for an item stratum, there is still a high probability that its price may ignore

[24]More information on hedonic methods in the CPI and the PPI can be found at https://www.bls.gov/cpi/quality-adjustment/questions-and-answers.htm and https://www.bls.gov/ppi/ppicomqa.htm.

[25]For all the quality adjustment methods used in the CPI and the PPI, see Chapter 17, https://www.bls.gov/opub/hom/pdf/cpi-20180214.pdf, of the BLS https://www.bls.gov/opub/hom/ and "Quality Adjustment in the Producer Price Index" at https://www.bls.gov/ppi/qualityadjustment.pdf, respectively.

[26]For detailed information on the construction of the CPI, see Chapter 17, https://www.bls.gov/opub/hom/pdf/cpi-20180214.pdf, of the BLS https://www.bls.gov/opub/hom/.

the overall quality change in that strata. According to Groshen, Moyer, Aizcorbe, Bradley, and Friedman (2017), in 2014, matches were found for 73% items in the CPI, the prices of which are not quality-adjusted. In turn, the components of GDP and outputs deflated by the prices of these matched items are highly likely to miss the change in quality.

With a large share of real GDP unadjusted for quality, the BEA data under-estimates the quality improvement and productivity gain from the return to input variety. Consequently, the amplification effects of extensive margin adjustments on the fluctuations of real GDP are under-estimated in the published data compared to those in my model.

Meanwhile, it is hard to know how much the amplification effect is under-estimated in the BEA data. As mentioned in Section 1, the return to variety may affect output in two ways. First, changes in the number of input varieties lead to changes in the efficiency of producing the same goods due to the division of labor as in Ethier (1982); Second, changes in the number of input varieties result in quality changes. In the first case, because the quantity rather than the quality changes, real GDP will reflect the productivity change. In the second case, however, real GDP will possibly miss quality changes. Since there is no data on either the relative weights of the two cases in the return to variety or the proportion of quality changes in the real GDP that is missed in the BEA data, I cannot conclude how much the BEA data can reflect the true amplification effects of extensive margin adjustments.